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# A Model for Industrial Real-Time Systems

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**Abstract.** Introducing automated formal methods for large industrial real-time systems is an important and actual research challenge. We propose timed process automata (TPA) for modeling and analysis of time-critical systems which can be open, hierarchical, and dynamic. The model offers two essential features for large industrial systems: (i) compositional modeling with reusable designs for different contexts, and (ii) automated state-space reduction technique. Timed process automata model dynamic networks of continuous-time communicating control processes which can activate other processes. We show how to automatically establish safety and reachability properties of TPA by reduction to solving timed games. To mitigate the state-space explosion problem, an automated state-space reduction technique using compositional reasoning and aggressive abstractions is also proposed.

## 1 Introduction

This paper develops a model for *automated analysis of safety and reachability* properties in large industrial *time-critical systems*. To fulfill industrial requirements, we consider time-critical systems that are open (communicate with external components), hierarchical (can be decomposed and recomposed into smaller control systems), and dynamic (the decomposition can change over time). In the paper, we use *real-time systems*, meaning time-critical systems that fulfill all these features. The model also facilitates compositional modeling and reusable designs for different contexts.

An *open system* continuously interacts with an unpredictable environment. A good example of time-critical open systems is a pacemaker, which continuously interacts with a heart, an uncontrolled environment. The pacemaker's performance crucially depends on the exact timing of an action performed either by the system or by the environment. The *theory of timed games* [1,2,3,4] is well-known in the research community for the analysis of time-critical open systems.

A *hierarchical system* is a hierarchical composition of smaller systems. An automotive system, developed by an *original equipment manufacturer (OEM)*, may be used in different models of cars. In this case, the system has a *controller* which helps the system adapt to different *environments* and cars. In other words, the system is an open system, which has two distinguished interacting segments: the controller and the environment. Typically, these systems consist of other smaller systems in a *hierarchical* structure. For instance, a system **Actuator** can

be a component of a larger system *Position*, while *Position* can be a component of another system *Brake-by-Wire*, and so on. Every component of a system has a specific set of tasks; for example, system *Brake-by-Wire* may use its component *Position* to perform some desired tasks in interaction with the environment, and *Brake-by-Wire* may also indirectly—through using *Position*—use its sub-component *Actuator* to perform some desired tasks in interaction with the environment.

A *dynamic system* is a hierarchical system whose components may change over time. Many hierarchical systems have dynamic characteristic, which is activating components only when needed. Dynamic behaviors are an important feature when resource constraints (such as limited memory) do not allow one to keep all the components active at the same time. Sometimes dynamic behaviors are inherent to the system. For example, we applied timed game theory in a GM project to construct a fault-tolerant framework for a hierarchical open system that has a scheduler, a set of tasks, and a set of subtasks; only the scheduler is active in the initial system-state; subtasks are activated by their parent tasks, and the top level tasks are activated by their scheduler; thus the scheduler controls tasks, and a task controls its subtasks; due to the termination or the initialization of tasks (or subtasks) the structures of the processes may change; thus the system is a dynamic open system [5].

*Timed automata* (TA) [6,7] are desirable for the development of real-time systems because TA can model and analyze both discrete-time controllable behaviors of the system and continuous-time uncontrollable behaviors of the environment. Timed automata and their more than 80 variants [8] are mostly studied for the development of embedded systems, where behaviors of the components are known and the number of the components is static. As a result, modeling techniques, automated analyses, and other key issues of TA are typically addressed for *static closed systems*. The application domain of TA is growing [8]. In our two projects with General Motors (GM), we used different TA-based analyses to investigate the fault-tolerance of real-time systems, which are part of many large-scale safety-critical systems. During our industrial projects, we observed that continuous-time formal methods of TA may provide the most accurate analysis; however, TA are not suited for industrial real-time systems mainly because of poor *scalability*. Moreover, we found that TA have no structured support for modeling real-time systems, which may lead to cumbersome design details in a large-scale real-time system having several control hierarchies. The paper extends TA to achieve better modeling support and scalability for automated analysis of real-time systems.

We propose *timed process automata* (TPA), a variant of TA, for the development of industrial real-time systems. The proposed variant provides compositional modeling (with reusable designs for different contexts) and automated analysis—a system needs to be modeled and analyzed using TPA only once when copies of it are used as independent systems or multiple components of a larger system or components of different larger systems or a combination of all previous scenarios. The contributions of this paper include:

1. Timed process automata, the first model that provides compositional modeling with reusable designs for dynamic hierarchical open time-critical systems.

2. Definition of a formal semantics for TPA.
3. An automated analysis for safety and reachability properties of TPA.
4. The first automated state-space reduction technique for time-critical systems, which can be dynamic, hierarchical, and open.

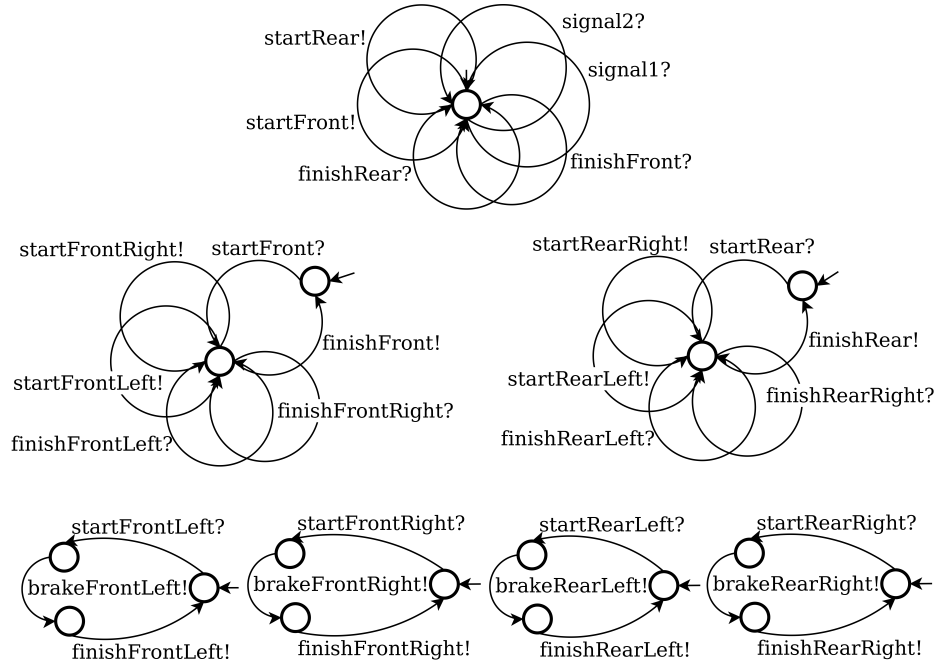
The rest of the paper can be divided in seven sections:

- Section 2** Describes the motivation for the work. The motivation is based on the experience achieved from a couple of automotive industrial projects.
- Section 3** Provides the required background to understand the paper.
- Section 4** Presents the syntax (Sect. 4.1) and the semantics (Sect. 4.2) of TPA, which use start actions, finish actions, final locations, and channels to facilitate compositional modeling to reuse designs without manual alterations.
- Section 5** Presents an automated analysis technique—based on timed games—for TPA. The analysis model of a timed process automaton  $T$  is constructed by composing a finite number of *timed I/O automata* (TIOA) [9,2,4], a variant of TA, to mimic the execution of  $T$ . The analysis model is constructed using an automated technique that allows the designer to avoid manual alteration techniques for different compositions. Other than the automated construction, the constructed analysis models essentially are TIOA models, whose state spaces are too large to analyze industrial real-time systems.
- Section 6** Develops an automated state-space reduction technique that converts each callee process into a small automaton having only two locations and two edges, irrespective of the size of the callee. The technique uses structured construction of TPA, compositional reasoning, aggressive abstractions, and fewer synchronizations to ensure smaller state space.
- Section 7** Discusses related work. It also classifies TPA depending on the classification of TA variants presented in a previous work [8].
- Section 8** Concludes the paper.

## 2 Motivation

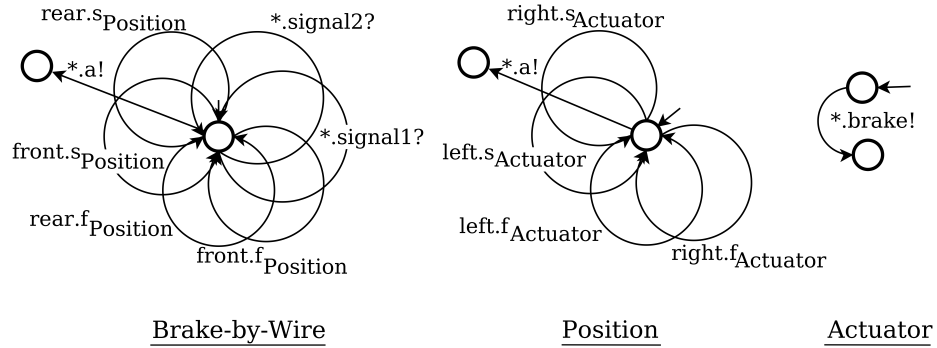
The first goal of the paper is to develop a real-time model, where a designer will not need to readjust a design for different compositions. The second—but the main—goal is to allow automated analysis of the model for industrial systems.

Figure 1 presents an abstract **Brake-by-Wire** system modeled using TIOA, and the system is developed by an OEM. The model has seven automata representing different copies of only three elements: one copy of the *main thread* of **Brake-by-Wire** (the top automaton), two copies of the main thread of **Position** (the two automata in the middle), and four copies of **Actuator** system (the four automata in the bottom). Each **Position** system contains two *children* (**Actuator** systems) and its main thread that schedules the children, communicates with its parent (the main thread of **Brake-by-Wire**), and performs some other functions, which cannot be performed by the children. Similarly, the **Brake-by-Wire** system contains two children (**Position** systems) and its main thread that schedules the children and performs some other functions, which cannot be performed by the children.



**Fig. 1.** An abstract Brake-by-Wire system modeled using standard TIOA

In this model, the main thread of Brake-by-Wire is the *root*, which does not have a parent. However, in the future a car manufacturer may include this Brake-by-Wire system in a car and then the main thread of Brake-by-Wire will no longer be the root. Then a central control system may be able to start the main thread of Brake-by-Wire. To analyze the new complex system, a designer will need to manually alter the model again by including *start* and *finish* actions (in the top automaton of Fig. 1). Let us assume a complex system contains  $N$  Break-by-Wire systems; to analyze this complex system, a designer will need to manually construct at least  $N \times 7$  automata with a proportionally growing alphabet! Existing TA-based modeling techniques do not support compositional modeling with reusable designs



**Fig. 2.** The same Brake-by-Wire system of Fig. 1 is modeled using TPA

for different contexts; that is, a design may need to be altered manually in every composition. All these ad hoc alterations may make a large industrial design incomprehensible and error-prone. Figure 2 presents the same **Brake-by-Wire** system of Fig. 1 modeled by using TPA. Timed process automata always model a system only once. For example, Fig. 2 presents only three TPA, which are equivalent to the seven automata of Fig. 1. Moreover, the number of copies and the root status of **Brake-by-Wire** system has no impact on the new design.

To the best of our knowledge, no automated state-space reduction technique has been developed for the analysis of real-time systems. During our two projects with GM, we noticed that even a (practically) very small real-time system may have a state space too large for automated formal analysis because of hierarchy, dynamic behaviors, and time calculations. We overcame the scalability problem in one of the projects—construction of a fault-tolerance framework [5]—by developing a manual state-space reduction technique that applies aggressive abstractions and uses less synchronizations. Applying this manual technique to a design of an industrial system is a challenging task. Moreover, the technique may not work for every real-time systems. A generalized automated reduction technique, therefore, is needed for analysis of large real-time systems, which is provided in this paper by presenting an automated reduction technique for TPA.

### 3 Background

The semantic construction of TA is expressed using semantics objects called *timed transition systems (TTS)* [10,4,7]. A *timed I/O automaton* [9,2,4] is a timed automaton which has an input alphabet along with a regular output alphabet. The controller plays controllable output transitions and the environment plays uncontrollable input transitions; thus TIOA are a natural model for timed games. Two TIOA are *well-formed* with each other if they don't have a common output action. The *composition of two well-formed TIOA* forms a larger timed I/O automaton [2,4]. The section defines TTS, TIOA, composition of TIOA, and all other terms required to understand the remaining paper.

**Definition 1** [10,4,7] A timed transition system is a tuple  $\mathcal{T} = (St, s_0, \Sigma, \dashrightarrow)$ , where  $St$  is an infinite set of states,  $s_0 \in St$  is the initial state,  $\Sigma$  is an alphabet, and  $\dashrightarrow: St \times (\Sigma \cup \mathbb{R}_{\geq 0}) \times St$  is a transition relation.

We use  $d \in \mathbb{R}_{\geq 0}$  to denote delay. A TTS satisfies *time determinism* (i.e., whenever  $s \xrightarrow{d} s'$  and  $s \xrightarrow{d} s''$  then  $s' = s''$  for all  $s \in S$ ), *time reflexivity* (i.e.,  $s \xrightarrow{0} s$  for all  $s \in S$ ), and *time additivity* (i.e., for all  $s, s'' \in S$  and all  $d_1, d_2 \in \mathbb{R}_{\geq 0}$  we have  $s \xrightarrow{d_1+d_2} s''$  iff there exists an  $s'$  such that  $s \xrightarrow{d_1} s'$  and  $s' \xrightarrow{d_2} s''$ ). A *run*  $\rho$  of a TTS  $\mathcal{T}$  from a state  $s_1 \in St$  is a sequence  $s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} s_3 \cdots \xrightarrow{a_n} s_{n+1}$  such that for all  $1 \leq m \leq n$ :  $s_m \xrightarrow{a_m} s_{m+1}$  with  $a_m \in \Sigma \cup \mathbb{R}_{\geq 0}$ . A state  $s$  is *reachable* in a transition system  $\mathcal{T}$  if and only if there is a run  $s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \cdots \xrightarrow{a_{n-1}} s_n$ , where  $s = s_n$ . *Timed I/O transition systems (TIOTS)* are TTS with input and output modalities on transitions. Timed I/O transition systems are used to define semantics of TIOA.

A *clock* is a non-negative real variable. A *constraint*  $\delta \in C(X, V)$  over a set of clocks  $X$  and over a set of non-negative finitely bounded integer variables  $V$  is generated by the grammar  $\delta ::= x_m < q \mid k < \alpha \mid x_m - x_n < q \mid \text{true} \mid \Phi \wedge \Phi$ , where  $q \in \mathbb{Q}_{\geq 0}$ ,  $\alpha \in \mathbb{Z}_{\geq 0}$ ,  $\{x_m, x_n\} \subseteq X$ ,  $k \in V$  and  $< \in \{<, \leq, >, \geq\}$ . Consequently, the set of *clock constraints*  $C(X)$  is the set of constraints  $C(X, V)$ , where  $V = \emptyset$ . Let  $\Psi(V)$  be the set of assignments over the set of variables  $V$ .

**Definition 2** [9,2,4,7] A timed I/O automaton is a tuple  $\mathcal{A} = (L, l_0, X, V, A, E, I)$ , where  $L$  is a finite set of locations,  $l_0 \in L$  is the initial location,  $X$  is a finite set of clocks,  $V$  is a finite set of non-negative finitely bounded integer variables,  $A = A_i \oplus A_o$  is a finite set of actions, partitioned into input actions  $A_i$  and output actions  $A_o$ ,  $E \subseteq L \times A \times \Phi(X, V) \times \Psi(V) \times 2^X \times L$  is a set of edges, and  $I : L \rightarrow C(X)$  is a total mapping from locations to invariants.

A *clock valuation* over  $X$  is a mapping  $\mathbb{R}_{\geq 0}^X : X \rightarrow \mathbb{R}_{\geq 0}$ . Given a clock valuation  $v$  and  $d \in \mathbb{R}_{\geq 0}$ , we write  $v + d$  for the clock valuation in which for each clock  $x \in X$  we have  $(v + d)(x) = v(x) + d$ . For  $\lambda \subseteq X$ , we write  $v[x \mapsto 0]_{x \in \lambda}$  for a clock valuation agreeing with  $v$  on clocks in  $X \setminus \lambda$ , and giving 0 for clocks in  $\lambda$ . For  $\phi \in \Phi(X, N)$  and  $v \in \mathbb{R}_{\geq 0}^X$ , we write  $v, N \models \phi$  if  $v$  and  $N$  satisfy  $\phi$ . Let  $e = (l, a, \phi, \theta, \lambda, l')$  be an edge, then  $l$  is the source location,  $a$  is the action label, and  $l'$  is the target location of  $e$ ; the constraint  $\phi$  has to be satisfied during the traversal of  $e$ ; the set of clocks  $\lambda \in 2^X$  are reset to 0 and the set of non-negative finitely bounded integer variables are updated to  $\theta$  whenever  $e$  is traversed.

**Definition 3** [2,4] Two timed I/O automata  $\mathcal{A}^m = (L^m, l_0^m, X^m, N^m, A^m, E^m, I^m)$  and  $\mathcal{A}^n = (L^n, l_0^n, X^n, N^n, A^n, E^n, I^n)$  are well-formed with each other when  $A_o^m \cap A_o^n = \emptyset$ ,  $X^m \cap X^n = \emptyset$ , and  $N^m \cap N^n = \emptyset$ ; when well-formed, their composition is a TIOA  $\mathcal{A} = \mathcal{A}^m \parallel \mathcal{A}^n = (L^m \times L^n, (l_0^m, l_0^n), X^m \cup X^n, N^m \cup N^n, A, E, I)$ , where  $A = A_i \cup A_o$  with  $A_o = A_o^m \cup A_o^n$  and  $A_i = (A_i^m \cup A_i^n) \setminus A_o$ . The set of edges  $E$  contains:

- $((l^m, l^n), a, \phi^m \wedge \phi^n, \lambda^m \cup \lambda^n, \theta^m \cup \theta^n, (l'^m, l'^n)) \in E$  for each  $(l^m, a, \phi^m, \theta^m, \lambda^m, l'^m) \in E^m$  and  $(l^n, a, \phi^n, \theta^n, \lambda^n, l'^n) \in E^n$  if  $a \in \{A_i^m \cap A_o^n\} \cup \{A_o^m \cap A_i^n\}$
- $((l^m, l^n), a, \phi^m, \lambda^m, \theta^m, (l'^m, l'^n)) \in E$  for each  $(l^m, a, \phi^m, \lambda^m, \theta^m, l'^m) \in E^m$  if  $a \notin A^n$
- $((l^m, l^n), a, \phi^n, \lambda^n, \theta^n, (l'^m, l'^n)) \in E$  for each  $(l^n, a, \phi^n, \lambda^n, \theta^n, l'^n) \in E^n$  if  $a \notin A^m$

and the set of invariants  $I$  is constructed as follows:  $I(l^m, l^n) = I^m(l^m) \wedge I^n(l^n)$

## 4 Processes

Timed process automata model processes, where each process is a real-time system. Every process hierarchically contains its active callee processes. Thus the control of a process is hierarchically shared with its active callee processes. The main thread of a process can activate callee processes via communication channels. An active process can receive any input in any state. An active callee process can deactivate itself in any state of the main thread of its caller process. An activated callee process dies within its worst-case execution time. This section presents the syntax and the semantics of TPA.

#### 4.1 Timed Process Automata

Timed process automata are a variant of TIOA. Unlike a timed I/O automaton, a timed process automaton has a finite set of *start actions*  $A_s$ , a finite set of *finish actions*  $A_f$ , a final location  $l_f$ , and a finite set of *channels*  $C$ .

The set of *actions*  $A = A_i \oplus A_o \oplus A_s \oplus A_f$  of a timed process automaton is a disjoint union of finite sets of input actions  $A_i$ , output actions  $A_o$ , start actions  $A_s$ , and finish actions  $A_f$ . For every set of actions  $A$ , there exists a bijective mapping between its start actions  $A_s$  and finish actions  $A_f$  in such a way that for each start action  $s_N \in A_s$  there is exactly one finish action  $f_N \in A_f$ , and vice versa. These actions can be used for starting and terminating processes associated with  $N$ . We use  $s$  and  $f$  with the name  $N$  (of another timed process automaton) as a subscript index (e.g.,  $s_N$  and  $f_N$ ) to denote a start action and a finish action, respectively. We use the same subscript to indicate *paired* actions. We write  $a$  to denote an action in general. Processes synchronize via instantaneous channels. Each TPA uses the same designated symbols for its *public channel*  $(*)$  and *caller channel*  $(\Delta)$ . We use  $c$  to denote a channel in general.

**Definition 4** A timed process automaton is a tuple  $T = (L, l_0, X, A, C, E, I, l_f)$ , where  $L$  is a finite set of locations,  $l_0 \in L$  is the initial location,  $X$  is a finite set of clocks,  $A = A_i \oplus A_o \oplus A_s \oplus A_f$  is a finite set of actions as described above,  $C$  is a finite set of channels,  $E \subseteq (L \times A \times C \setminus \{\Delta, *\} \times \Phi(X) \times 2^X \times L) \cup (L \times (A_i \cup A_o) \times \{\Delta, *\} \times \Phi(X) \times 2^X \times L)$  is a set of edges,  $I : L \rightarrow \Phi(X)$  is a total mapping from locations to invariants, and  $l_f \in L$  is a designated final location which does not have any outgoing edges to other locations and has the invariant  $I(l_f) = \text{true}$ .

Figure 2 presents TPA Brake-by-Wire, Position, and Actuator. In the figure, initial locations have a dangling incoming edges, final locations are filled with black, and TPA names are underlined. The final location  $l_f$  of a TPA may be unreachable from the initial location (and then  $l_f$  is not shown in the figure).

#### 4.2 Process Executions

Every instance of a timed process automaton is a *process*. Two processes of the same timed process automaton represent two different copies of the same system. Every process has a unique *process identifier*. A *process* is a tuple  $P = (\text{id}(P), \text{tpa}(P), \text{channel}(P))$ , where  $\text{id}(P)$ <sup>3</sup> is the process identifier, timed process automaton  $\text{tpa}(P)$  defines the execution logic, and *caller channel*  $\text{channel}(P)$  is the private channel to communicate with the caller and the other processes which are started via the same channel. A process  $Q$  is a *callee* of  $P$  if  $P$  is the caller of  $Q$ . We use  $\perp$  to denote the caller channel of the root process. Every process  $P$  of  $\text{tpa}(P) = (L, l_0, X, A, C, E, I, l_f)$  has its own copy  $P.c$  of channel  $c \in C$ . We write  $P.c.a$  meaning that action  $a$  is performed via channel  $P.c$ .

At the same time, no two processes of the same timed process automaton can have the same caller channel. A process  $P$ , therefore, may have at most  $|C| \times |A_s|$

<sup>3</sup> To avoid clutter, we abuse notation by writing  $P$  instead of  $\text{id}(P)$ .



active callee processes. For example, an instance of automaton **Brake-by-Wire** of Fig. 2 can activate at most two instances of automaton **Position** of Fig. 2 at the same time via two different channels **front** and **rear**, where the instance of **Brake-by-Wire** is the caller process of the two instances of **Position**, which are the callee processes of the instance of **Brake-by-Wire**. A *subprocess* is a callee or a callee of a subprocess, recursively. For example, every instance of **Brake-by-Wire** has six subprocesses: two instances of **Position** and four instances of automaton **Actuator** of Fig. 2. Every process hierarchically contains all of its subprocesses. Two processes are *siblings* if they have the same caller channel. The caller can use separate channels to differentiate control over different callees, even if they are processes of the same automaton.

A process  $P$  starts a process  $Q$  of an automaton  $\text{tpa}(Q)$  via channel  $P.c$  by traversing an edge  $e_1 = (\neg, \text{stpa}(Q), c, \neg, \neg)$  labeled by a start action  $\text{stpa}(Q)$  if there exists no active process of  $\text{tpa}(Q)$  with caller channel  $P.c$ ; dually,  $P$  traverses an edge  $e_2 = (\neg, \text{jtpa}(Q), c, \neg, \neg)$  labeled by the paired finish action  $\text{jtpa}(Q)$  whenever  $Q$  reaches its final state. No edge labeled by  $\text{jtpa}(Q)$  will ever be traversed if  $\text{tpa}(Q)$  is a *non-terminating timed process automaton*. Correspondingly, note that existing processes may start different processes of  $\text{tpa}(Q)$ —but always with different process identifiers. However, only  $P$  listens to finish action  $\text{jtpa}(Q)$  via channel  $\text{channel}(Q)$ . Process  $P$  traverses an edge  $e = (\neg, a, c, \neg, \neg)$  when  $P$  receives (resp., sends) an input (resp., output)  $a$  in channel  $P.c$ . Process  $P$  communicates with its callee  $Q$  via  $\text{channel}(Q)$  and with the environment via channel  $P.*$ .

We formalize the above mechanics of execution by first giving the semantics of the main thread of the process, ignoring its subprocesses in Def. 5 and then giving the semantics of the entire process in Def. 6. The standalone semantics of a process is essentially the same semantics of a standard TIOA [7,9,2,4]. The main difference is that states are decorated with process identifiers and edges with channel names to distinguish different instances of the same TPA in Def. 6. Also the caller channel  $\Delta$  is instantiated for an actual parent process. The technical reason for this will become apparent in Def. 6.

**Definition 5** *The standalone semantics  $\mathcal{S}[[P]]$  of a process  $P = (P, \text{tpa}(P), \text{channel}(P))$  is a TIOTS  $\mathcal{S}[[P]] = (L \times \mathbb{R}_{\geq 0}^X \times P, (l_0, \mathbf{0}, P), A^P, \rightarrow)^4$ , where  $\text{tpa}(P) = (L, l_0, X, A, C, E, I, I_f)$ ,  $\mathbf{0}$  is a function mapping every clock to zero and  $\rightarrow \subseteq (L \times \mathbb{R}_{\geq 0}^X \times \{P\}) \times (A^P \cup \mathbb{R}_{\geq 0}) \times (L \times \mathbb{R}_{\geq 0}^X \times \{P\})$  is the transition relation generated by the following rules:*

*Action For each clock valuation  $v \in \mathbb{R}_{\geq 0}^X$  and each edge  $(l, a, c, \phi, \lambda, l') \in E$  such that  $v \models \phi$ ,  $v' = v[x \mapsto 0]_{x \in \lambda}$ , and  $v' \models I(l')$  we have  $(l, v, P) \xrightarrow{P.c.a} (l', v', P)$  if  $c \neq \Delta$ , otherwise  $(l, v, P) \xrightarrow{\text{channel}(P).a} (l', v', P)$*

*Delay For each clock valuation  $v \in \mathbb{R}_{\geq 0}^X$  and for each delay  $d \in \mathbb{R}_{\geq 0}$  such that  $(v + d) \models I(l)$  we have  $(l, v, P) \xrightarrow{d} (l, v + d, P)$ .*

The transition system induced by the standalone semantics of a process is time deterministic, time reflexive, and time additive.

<sup>4</sup>  $A^P$  is the set of actions where action names are constructed using regular expression  $(P^c \cdot C \mid \text{channel}(P))^c \cdot A$ .

*Ground timed process automata* are TPA that cannot perform a start or finish action ( $A_s \cup A_f = \emptyset$ ). Automaton **Actuator** in Fig. 2, for instance, is a ground timed process automaton. *Compound timed process automata* are TPA that can perform a start or finish action ( $A_s \cup A_f \neq \emptyset$ ). For example, **Brake-by-Wire** and **Position** in Fig. 2 are compound TPA. A *well-formed channel* cannot be used by two processes sharing an output action. Processes of a *well-formed timed process automaton* have only well-formed channels. Non-recursive TPA are defined inductively using the following rules: (i) every ground timed process automaton is a non-recursive timed process automaton, and (ii) a compound timed process automaton which performs only those start and finish actions whose subscripts are the names of some other existing non-recursive TPA is a non-recursive timed process automaton. All three TPA in Fig. 2, for example, are non-recursive TPA. A process of a non-recursive timed process automaton hierarchically contains only a finite number of subprocesses. The caller may activate an idle process, iteratively. Thus a process may activate a subprocess an arbitrary number of times. In this paper, we are only concerned with non-recursive well-formed TPA.

A *standalone final state* of a process  $P$  is  $(l_f, v, P)$ , where  $v$  is any clock valuation. We use  $st^P$ ,  $st_i^P$ ,  $c^P$ , and  $st_f^P$  to denote a standalone state, the standalone initial state, the set of channels, and a standalone final state of process  $P$ , respectively. We say that a process  $P$  is  $A'$ -enabled for a channel  $P.c$  if for every reachable standalone state  $st^P$  we have  $st^P \xrightarrow{P.c.a} st'^P$  for some standalone state  $st'^P$  for each action  $a \in A'$ . We require that each process  $P$  is  $A_i$ -enabled (input enabled) for all channels of  $P$ , and  $A_f$ -enabled (finish enabled) for all channels of  $P$  other than channels  $P.\Delta$  and  $P.*$  to reflect the phenomenon that inputs from the environment and the deaths of callees are independent events, beyond the control of a process. We present the semantics of a process in the following:

**Definition 6** *The global operational semantics  $\mathcal{G}[P]$  (semantics  $\llbracket P \rrbracket$  for short) of a process  $P = (P, \text{tpa}(P), \perp)$  is a TIOTS  $\mathcal{G}[P] = (2^S, s_0, \mathbb{P} \times \mathbb{C} \times \mathbb{A}, \rightarrow)$ , where  $S$  is the set of all the standalone states of all the processes in the universe,  $\text{tpa}(P) = (L, l_0, X, A, E, I, l_f)$ ,  $s_0 = \{st_0^P\}$  is the initial state,  $\mathbb{P}$  is the set of all the processes in the universe,  $\mathbb{C}$  is the set of all the channels in the universe,  $\mathbb{A}$  is the set of all the actions in the universe, and  $\rightarrow \subseteq 2^S \times (\mathbb{P} \times \mathbb{C} \times \mathbb{A} \cup \mathbb{R}_{\geq 0}) \times 2^S$  is the transition relation generated by the following rules:*

$$\begin{array}{c}
\frac{st^Q \xrightarrow{Q.c.s_T} st'^Q \text{ and } c \notin \{\Delta, *\} \quad \{st^W \in s \mid \text{channel}(W) = Q.c \text{ and } \text{tpa}(W) = T\} = \emptyset}{st^Q \in s \quad (R, T, Q.c) \text{ is a freshly started process}} \text{ START} \\
\frac{}{s \xrightarrow{Q.c.s_T} s \setminus \{st^Q\} \cup \{st_0^R, st'^Q\}} \\
\\
\frac{st_f^R, st^Q \in s \text{ and } \text{channel}(R) = Q.c \quad \{st^U \in s \mid \text{channel}(U) \in C^R\} = \emptyset \quad st^Q \xrightarrow{Q.c.j_{\text{tpa}(R)}} st'^Q}{s \xrightarrow{Q.c.j_{\text{tpa}(R)}} s \setminus \{st_f^R, st^Q\} \cup \{st'^Q\}} \text{ FINISH} \\
\\
\frac{s' = \{st'^Q \mid st^Q \xrightarrow{d} st'^Q \text{ and } st^Q \in s \text{ and } (st^Q \neq st_f^Q \text{ or } |s| = 1)\} \quad |s| = |s'|}{s \xrightarrow{d} s'} \text{ DELAY}
\end{array}$$

$$\begin{array}{c}
\frac{a \notin \bigcup_{st^Q \in s} A_o^{\text{tpa}(Q)} \quad s' = \{st^Q \in s \mid st^Q \xrightarrow{Q,*.a} st'^Q\}}{s \xrightarrow{a} s \setminus s' \cup \{st'^Q \mid st^Q \xrightarrow{Q,*.a} st'^Q \text{ and } st^Q \in s\}} \text{ INPUT} \\
\\
\frac{s'^Q \xrightarrow{W.c.a} st'^Q \text{ and } a \in A_o^Q \text{ and } st^Q \in s \quad s' = \{st^R \in s \mid st^R \xrightarrow{W.c.a} st'^R \text{ and } W.c \text{ is a channel}\}}{s \xrightarrow{Q.c.a} s \setminus s' \cup \{st'^R \mid st^R \xrightarrow{W.c.a} st'^R \text{ and } st^R \in s\}} \text{ OUTPUT}
\end{array}$$

A *global state* is a set which holds standalone states of all active processes. The START rule states that the initial standalone state of a freshly started callee is added to the global state whenever the corresponding start action is performed by its caller. The rule also states that no two active processes can have the same timed process automaton and the same caller channel. The FINISH rule prescribes that the standalone-final state of a callee is removed from the global state and the caller executes the corresponding finish action whenever that callee is in the standalone-final state and no standalone state of its subprocesses is in global state. Thus the rule defines *global-final state* (*final state* for short) of a process: a process is in its the final state when the process is in its final location and the process has no active subprocess. The DELAY rule declares that globally a process can delay if that process and all of its active subprocesses can delay in their respective standalone semantics. Every subprocess is a part of the root process and thus if a subprocess is performing an action (or not idle) then the root process is also not idle. The rule also says that a process cannot delay if that process or any of its subprocess is in its global final state. That means a process finishes as soon as it reaches its final state. The INPUT rule states that a process receives an input from the environment via channel *id.\**. Rule OUTPUT declares a process send an output via channel *id.c* to others who share *id.c*. It follows from the properties of the standalone semantics that the transition system induced by Def. 6 is time deterministic, time reflexive, and time additive. The process semantics, therefore, defines a well-formed TIOTS. This allows us to use TA as a basis for analyzing TPA. A *local run* of the main thread of a process  $P$  is a standalone run of  $P$  for which there exists a global run of  $P$  such that every transition of that standalone run occurs in that global run. The *local behavior* of the main thread of  $P$  consists of all of its local runs.

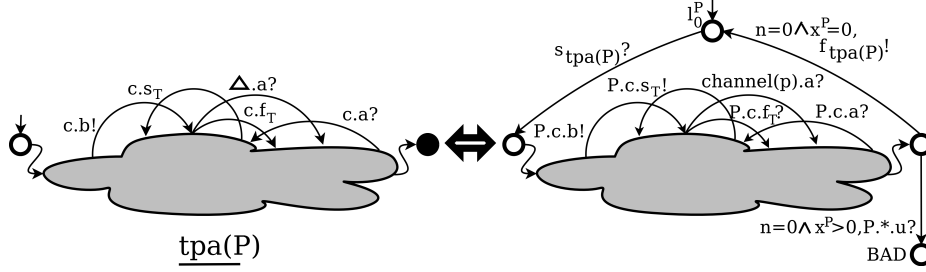
## 5 Analysis

We are interested in *safety* and *reachability* properties of real-time systems. This section explains how such analyses can be performed using the theory of timed games. A standard timed I/O automaton can be viewed as a concurrent two-player timed game, in which the players decide both which action to play, and when to play it. The input player represents the environment, and the output player represents the system itself. Similarly, the main thread of a process acts as a concurrent two-player timed game: the environment plays input transitions and finish transitions, and the main thread of the process plays output transitions and

start transitions. Let's consider interactions of a process defined in the previous section. A process controls its output and start transitions. After starting a callee, the main thread of the caller knows that the paired finish action will arrive within the worst-case execution time of the associated callee. However, the main thread does not have any control on the exact arrival time of a finish action. Finish transitions along with input transitions are uncontrollable. The environment of the main thread of a process consists of all the connected processes (such as caller, siblings, and subprocesses) and all unconnected entities.

A global state of a process is safe if and only if all of the standalone states which it contains are safe locations. A *safety property* asserts that the system remains inside a set of global-safe states regardless of what the environment does. We are interested in *Safety Property I*: *Given a process  $P$  and a set of unsafe locations  $L_U$  of  $P$ , can the controller avoid  $L_U$  in  $P$  regardless of what the environment does?* A global state of a process is a target state if and only if at least one of its standalone states contains a target location. A *reachability property* asserts that the system reaches any of the global-target states regardless of what the environment does. We are interested in *Reachability Property I*: *Given a process  $P$  and a set of target locations  $L_T$  of  $P$ , can the controller reach a location of  $L_T$  in  $P$  regardless of what the environment does?*

The monolithic analysis constructs a static network of automata to represent all possible global executions by mimicking the hierarchical call tree of the analyzed process. It simulates a process execution by changing states of pre-allocated TIOA which fall into two groups: a *root automaton* to simulate the local behaviors of the main thread of the root process and a finite set of *standalone automata* to simulate the local behaviors of the main threads of the subprocesses.



**Fig. 3.** A generalized view of standalone automata construction

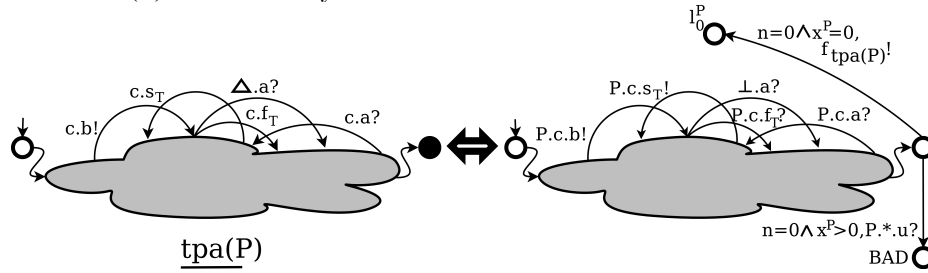
*Standalone Automata* We construct a standalone automaton for each subprocess to simulate the main thread of that process. To construct a standalone automaton, we prefix the timed process automaton with a simulated start action and suffix it with a simulated finish action. We use non-negative finitely bounded integer variables<sup>5</sup> in standalone automata to count the number of active callees, in order to

<sup>5</sup> The use of non-negative finitely bounded integer variables can be avoided if a more cumbersome encoding is used.

detect termination. We rename actions (resp.,  $a$ ) of processes uniformly to encode channel names (resp.,  $P.c$ ) in action names (resp.,  $P.c.a$ ) of standalone automata; because standard TIOA do not support private channels. A standalone automaton includes all the locations and slightly altered edges of the corresponding timed process automaton. Moreover, each standalone automaton has two additional locations: a new initial location  $l_0^P$  to receive (resp., send) a start (resp., finish) message from (resp., to) the caller, and a new unsafe location **BAD** to prevent the automaton from waiting in final states instead of finishing. Every start (resp., finish) increments (resp., decrements) a counter variable  $n$ . The automaton represents finishing of the process in the final location when  $n = 0$ . Formally, the standalone automaton of process  $P$  is  $\text{standalone}(P) = (L \cup \{l_0^P, \text{BAD}\}, l_0^P, X \cup \{x^P\}, \{n\}, A^P, E^P, I^P)$ , where  $\text{tpa}(P) = (L, l_0, X, A, C, E, I, l_f)$ ,  $l_0^P$  and **BAD** are two newly added locations,  $x^P$  is a newly added clock,  $n$  is a non-negative finitely bounded integer variable with the initial value 0,  $A_o^P = A_o' \cup A_s' \cup \{\text{channel}(P).f_{\text{tpa}(P)}\}$  and  $A_i^P = A_i' \cup A_f' \cup \{\text{channel}(P).s_{\text{tpa}(P)}, P.*.u\}$  such that  $A_m' = \{\text{channel}(P).a \mid a \in A_m\} \cup \{P.c.a \mid a \in A_m \text{ and } c \in C \setminus \{\Delta\}\}$  where  $m \in \{o, s, i, f\}$  and newly added actions are  $\text{channel}(P).s_{\text{tpa}(P)}$ ,  $\text{channel}(P).f_{\text{tpa}(P)}$ , and  $P.*.u$ . The set of edges  $E^P$  contains

- Converted edges that do not communicate via caller channel  $\Delta$ :
  - An edge  $(l, P.c.a, \phi, \xi, \lambda \cup \lambda', l') \in E^P$  for each edge  $(l, a, c, \phi, \lambda, l') \in E$ , where  $c \in C \setminus \{\Delta\}$ , the integer assignment is empty  $\xi = \emptyset$  when  $a \in A_o \cup A_i$ ,  $\xi = \{n - \}$  when  $a \in A_f$ , and  $\xi = \{n + \}$  when  $a \in A_s$
- Converted edges that communicate via caller channel  $\Delta$ :
  - An edge  $(l, \text{channel}(P).a, \phi, \emptyset, \lambda \cup \lambda', l') \in E^P$  for each edge  $(l, a, \Delta, \phi, \lambda, l') \in E$
- Additional new edges that simulate activation and deactivation:
  - Three more edges  $(l_0^P, \text{channel}(P).s_{\text{tpa}(P)}, \emptyset, \emptyset, X, l_0)$ ,  $(l_f, \text{channel}(P).f_{\text{tpa}(P)}, n = 0 \wedge x^P = 0, \emptyset, \emptyset, l_0^P)$ ,  $(l_f, P.*.u, n = 0 \wedge x^P > 0, \emptyset, \emptyset, \text{BAD})$  are in  $E^P$

$\lambda' = \emptyset$  when  $l' \neq l_f$ , otherwise  $\lambda' = \{x^P\}$ . The invariant function  $I^P$  maps each location  $l \in L$  to  $I(l)$  and maps each location  $l \in \{l_0^P, \text{BAD}\}$  to *true*. The standalone semantics of automaton  $\text{tpa}(P)$  and the semantics of standalone automaton  $\text{standalone}(P)$  are essentially the same.



**Fig. 4.** A generalized view of root automata construction

*Root Automata* To analyze a timed process automaton  $\text{tpa}(P) = (L, l_0, X, A, C, E, I, l_f)$ , we construct the root automaton  $\text{root}(P)$  of process  $P$ . Standalone automaton  $\text{standalone}(P)$  is slightly different from  $\text{root}(P)$ . The differences are: (i) the caller channel is always  $\perp$ , (ii) the initial location of root automaton  $\text{root}(P)$  is the

location  $l_0$ , which is also the initial location of  $\text{tpa}(P)$ , and (iii) root automaton does not have edge  $(l_0^P, \perp.\text{s}_{\text{tpa}(P)}, \emptyset, \emptyset, X, l_0)$ , which simulates activation of  $P$ .

*Monolithic Analysis Model* The monolithic analysis model of a ground timed processes automaton (such as **Actuator**) is its root automaton. We construct the monolithic analysis model of automaton  $\text{tpa}(P)$  in the following iterative manner:

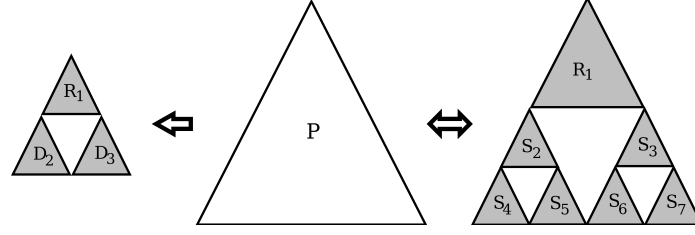
First Step: We construct the root automaton  $\text{root}(P)$ .

Iterative Step: We construct a standalone automaton for each triple  $(Q, s_T, c)$ , where  $Q$  is process for which we have constructed a standalone automaton or the root automaton,  $\text{tpa}(Q) = (L, l_0, X, A, C, E, I, l_f)$ ,  $c \in C \setminus \{\Delta, *\}$ ,  $s_T \in A_s$ , and  $(\rightarrow, s_T, c, \rightarrow, \rightarrow) \in E$ .

Figures 3–4 present a generalized view of standalone and root automata construction (Appendices A.1–A.3 present monolithic analysis models of processes of TPA **Actuator**, **Position**, and **Brake-by-Wire**). The monolithic analysis model constructs a parallel composition of all the TIOA constructed above. The construction is finite, and the composition is a timed I/O automaton; because we consider only non-recursive well-formed TPA. The created composition is timed-bisimilar to the global semantics (modulo hiding the special actions and renaming the others). Executions of this composition, when projected on the original alphabet are identical to the executions of the global semantics. Thus the composition has the same properties. We convert Safety Property I to *Safety Property II*: *Given a process  $P$  and a set of unsafe locations  $L_U$  of  $P$ , can the controller avoid  $L_U$  and all the BAD locations in the analysis model regardless of what the environment does?* We also convert Reachability Property I to *Reachability Property II*: *Given a process  $P$  and a set of target locations  $L_T$  of  $P$ , can the controller reach a location of  $L_T$  in the analysis model avoiding all the BAD locations regardless of what the environment does?* Avoiding all the newly added **BAD** locations in the analysis model ensures that each caller process performs corresponding finish action as soon as the callee finishes. Therefore, if a Safety Property I (resp., Reachability Property I) holds for a process then its corresponding Safety Property II (resp., Reachability Property II) also holds, and vice versa.

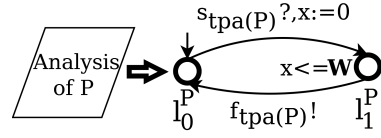
## 6 State-Space Reduction

We introduce an automated state-space reduction technique for TPA to counteract state-space explosion. The technique relies on compositional reasoning, aggressive abstractions, and reducing process synchronizations. In the monolithic analysis of Sect. 5, a callee can be represented by an arbitrary number of standalone automata and each of these automata can be arbitrarily large. The compositional reasoning replaces hierarchical trees of standalone automata representing subprocesses with simple abstractions (Fig. 5)—so called *duration automata*.

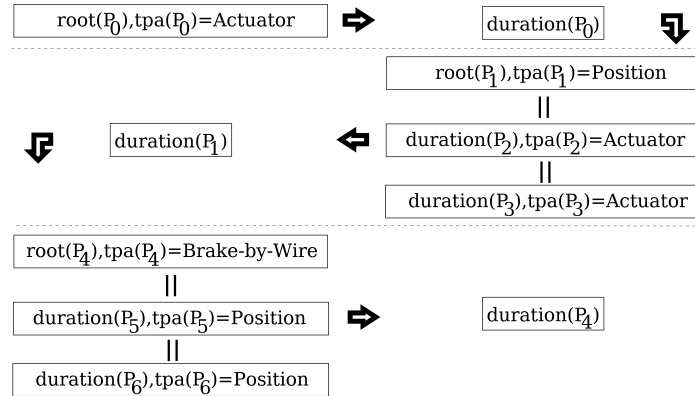


**Fig. 5.** A compositional analysis model (in the left) and a monolithic analysis model (in the right) of automaton *Brake-by-Wire*, where  $P$  is a process of the automaton,  $R_1$  is the root automaton,  $S_1$ – $S_7$  are standalone automata, and  $D_1$ – $D_2$  are duration automata

*Duration Automata* A duration automaton (Fig. 6) is timed I/O automaton with only two locations: the initial location ( $l_0^P$ ) and the active location ( $l_1^P$ ). A duration automaton of an analyzed process abstracts all the information of global executions of the process other than its *worst-case execution time (WCET)*. It can capture safety and reachability properties of interest. The *minimal-time safe reachability* of a target location is the *minimal-time reachability* [11,12] for which the controller has a winning strategy to reach that target location by avoiding unsafe states. We assume that the WCET  $\mathbf{W}$  of a process  $P$  is the minimal-time safe reachability time to reach location  $l_0^P$  of automaton  $\text{root}(P)$  in the analysis model of  $P$ . This is a known technique to limit the WCET of a controller [13,14]. The WCET of  $P$  is unknown ( $\mathbf{W} = \infty$ ) when it does not have a winning strategy for the minimal-time safe reachability to reach location  $l_0^P$  of  $\text{root}(P)$ . The duration automaton of process  $P$  is  $\text{duration}(P) = (\{l_0^P, l_1^P\}, l_0^P, \{x^P\}, \emptyset, A^P, E^P, I^P)$ , where  $\text{tpa}(P) = (L, l_0, X, A, C, E, I, l_f)$ ,  $A_i^P = \{\text{channel}(P).s_{\text{tpa}(P)}\}$ ,  $A_o^P = \{\text{channel}(P).j_{\text{tpa}(P)}\}$ , the set of edges  $E^P = \{(l_0^P, \text{channel}(P).s_{\text{tpa}(P)}, \emptyset, \emptyset, \{x^P\}, l_1^P), (l_1^P, \text{channel}(P).j_{\text{tpa}(P)}, \emptyset, \emptyset, \emptyset, l_0^P)\}$ , invariant  $I^P$  maps location  $l_0^P$  to *true*, and  $I^P$  maps location  $l_1^P$  to  $x^P \leq \mathbf{W}$ .



**Fig. 6.** A generalized view of duration automata construction



**Fig. 7.** Steps of the compositional analysis of automaton *Brake-by-Wire*

*Compositional Analysis Model* We construct the compositional analysis model in a bottom-up manner: analysis of a compound process is performed only after analyzing all its callees. Like the monolithic analysis, the compositional analysis model of a ground timed process automaton  $\text{tpa}(Q)$  (such as **Actuator**) is a root automaton of process  $Q$ . That TIOA is analyzed to construct a duration automaton of  $Q$ . For a compound process  $P$ , we analyze automaton  $\text{root}(P)$  in the context of the duration automata of its callees (instead of the entire hierarchical structure of subprocesses). We construct the compositional analysis model of a timed process automaton  $\text{tpa}(P)$  in the following manner:

First Step: We construct the root automaton  $\text{root}(P)$ .

Second Step: We construct a duration automaton for each triple  $(P, s_T, c)$ , where  $\text{tpa}(P) = (L, l_0, X, A, C, E, I, l_f)$ ,  $c \in C \setminus \{\Delta, *\}$ ,  $s_T \in A_s$ , and  $(\_, s_T, c, \_, \_) \in E$ .

Figure 7 presents the compositional analysis procedure of **Brake-by-Wire** (Appendices A.4–A.6 present the detailed models). The compositional model construction procedure terminates, and the composition of all the above TIOA is a timed I/O automaton; because we consider only non-recursive well-formed TPA. The duration automaton of a process can capture safety properties of interest: if a process has a winning strategy for a safety game, then all locations of its duration automaton are considered safe; otherwise, the active location ( $l_1^d$ ) of the duration automaton is added to the set of unsafe locations  $L_U$ . Now this duration automaton can be used as a sound context to analyze the caller automaton for safety. A safety property holds for a compound process when the main thread of the process preserves the property locally and allows the activation of a callee only if that callee also preserves the property. Duration automata can also capture reachability properties of interest: if a process has a winning strategy for a reachability game then the active location ( $l_1^d$ ) of the duration automaton is added to the set of target locations  $L_T$ ; otherwise, no target location is specified for this callee. This duration automaton can be used as a sound context to analyze the caller automaton for reachability. A reachability property holds for a compound process when the main thread of the process can reach the target locally or can activate a callee where the property holds. Like monolithic analysis, compositional analysis is performed for Safety Property II and Reachability Property II. The compositional analysis is sound: if a safety or reachability property holds in compositional analysis then it holds in the global semantics. A duration automaton does not contain any input and output actions of its process. Hence, the root automaton in a compositional model does not synchronize with the input and output actions of its callees—instead the automaton synchronizes for those actions with the environment. The duration automaton was created under the assumption that inputs are uncontrollable, so ignoring synchronization with inputs is sound. Similarly, it is sound to open the inputs of the root automaton from a callee, as they will be treated as open actions, so will be analyzed in a more hostile environment, than before the abstraction. Therefore, if a property holds in the compositional analysis then it also holds for the monolithic analysis.



*Scalability* In all the steps of Fig. 7, the largest composition contains only three automata, and except for the root automaton all are tiny duration automata. Monolithic analysis model of **Brake-by-Wire** is a composition of seven automata (App. A.3). A duration automaton is always small constant size (modulo the size of the WCET constant), and so also its state space is very simple, actually the discrete state space is independent of the input model as well. We applied our approach to examples like the one presented in our previous work [5]. First, we modeled that problem with standard TIOA using shared variables. Timed games solver Uppaal Tiga [15] produced  $\approx 290$  MB winning strategy for a safety objective for a configuration (C1 of [5])—a combination of different system parameters—in the TIOA model within  $\approx 94$  seconds. After that, we modeled the same system with TPA, and apply the state-space reduction technique. The same solver for the same configuration produced  $\approx 100$  KB winning strategy for the same objective in our compositional model within  $\approx 0.3$  seconds. Experiments for different configurations for the same system (of [5]) revealed that speed up of two orders of magnitude is possible with the compositional technique, while maintaining enough precision to obtain useful strategies for realistic scheduling problems. The size of composition in the monolithic analysis is exponential in the depth of the hierarchy, due to a product construction (it is also linear in the multiplication of sizes of all included standalone automata). In compositional analysis, the depth of the hierarchy is constant (only two layers) and we only take a product of one root automaton with several constant size duration automata; this explains why the practically obtained speed ups are so dramatic.

The efficiency gains are primarily due to the coarse abstraction of safety and reachability properties of an arbitrary large callee into a tiny duration automaton. Abstraction and compositional reasoning together might provide similar speed ups for TIOA [5]; however, restrictions that TPA impose on models allow to automate the procedure.

## 7 Related Work

Classical TA [6,7] and timed I/O automata [2,4] have explicit modeling support only for static non-hierarchical structures. In 2011, we identified and classified eighty variants of TA into eleven classes in a survey [8] and there may be many more. Timed process automata fall in the class of *TA with resources* [8] because of its ability to model dynamic behaviors, which is required when resource constraints do not permit one to activate all the components at the same time. More precisely, the model is a direct generalization of *task automata* [16], *dynamic networks of TA* [17], and *callable timed automata* [18]. These three variants model only closed systems, while TPA can model both closed and open systems. Task automata model only two layers (a scheduler and its tasks) of hierarchy, while TPA, dynamic networks of TA [17], and callable timed automata are able to model any numbers of hierarchies. Unlike TPA, none of them supports private communication, and none of them aimed for the automated formal development of large industrial real-time systems. In other words, none of these three models

provides compositional modeling with reusable designs for different contexts, and supports automated state-space reduction technique.

Dynamic networks of continuous-time automata have also been studied in the context of hybrid automata [19,20]. These works model physical environments using differential equations, which restrict the kinds of environments that can be described. In practice, large differential equations make analyses unmanageable, or can only give statistical guarantees [20]. These works focused on system dynamics, and did not support private communication. Timed process automata can be considered as a member of the class of *TA with succinctness* [8] because TPA hide many design details from the designers to achieve succinctness (like *TA variants with urgency* [21,22,8]). Timed process automata are also timed game automata [1,3,2,4] because the new variant uses timed games for analysis.

## 8 Conclusion

We have presented timed process automata that captures dynamic activation and deactivation of continuous-time control processes and private communication among the active processes. We have provided a safety and reachability analysis technique for non-recursive well-formed timed process automata. We have also designed an abstraction- and compositional reasoning-based state-space reduction technique for automated analysis of large industrial systems. Our analysis techniques can be applied in practice using any standard timed games solver such as Uppaal Tiga [15] and Synchia [23]. To the best of our knowledge, no prior work on dynamic network of timed automata considered private communication and open systems. This is also the first work that provides two important features for industrial time-critical dynamic open systems development: (i) compositional modeling with reusable designs for different contexts and (ii) automated state-space reduction technique.

It would be interesting to consider a model transformation from subset of *real-time  $\pi$ -calculus* [24,25] to TPA. This transformation might enable controllability analysis of  $\pi$ -calculus for open systems. The converse reduction from TPA to real-time  $\pi$ -calculus could also give several advantages: understanding TPA semantics in terms of the well-established  $\pi$ -calculus formalism, access to tools developed for real-time  $\pi$ -calculus [24], which might permit the analysis of recursive processes; it would also give a familiar automata-like syntax to  $\pi$ -calculus formalisms. It would also be relevant to minimize the number of subprocesses in controller synthesis. One may consider synthesis under this objective in future, possibly by reduction to *priced/weighted timed automata* [26,27].

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## A Sample I/O

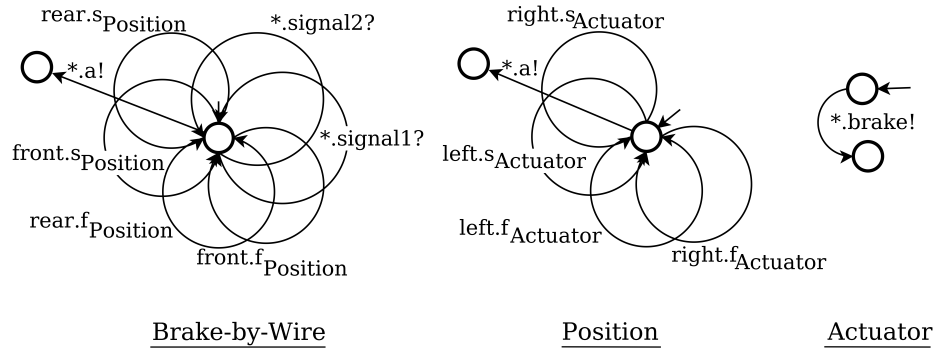


Fig. 8. The Brake-by-Wire system

### Input: timed process automaton **Actuator** creation

1. Name: Actuator
2. Input Actions:  $\emptyset$
3. Output Actions: {Brake}
4. Callees:  $\emptyset$
5. Clocks:  $\emptyset$
6. Channels:  $\emptyset$
7. Locations:  $l0, l1$
8. Initial Location:  $l0$
9. Final Location:  $l1$
10. Invariant:  $l0 : true, l1 : true$
11. Edges:
  - New edge:
    - Source Location:  $l0$
    - Action: brake
    - Channel:  $*$
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l1$

### Input: timed process automaton **Position** creation

1. Name: Position
2. Input Actions:  $\emptyset$

3. Output Actions: {a}
4. Callees: Actuator
5. Clocks:  $\emptyset$
6. Channels: {right, left}
7. Locations:  $l_0, l_1$
8. Initial Location:  $l_0$
9. Final Location:  $l_1$
10. Invariant:  $l_0 : true, l_1 : true$
11. Edges:
  - New edge:
    - Source Location:  $l_0$
    - Action: a
    - Channel: \*
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l_1$
  - New edge:
    - Source Location:  $l_0$
    - Action:  $s_{\text{Actuator}}$
    - Channel: right
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l_0$
  - New edge:
    - Source Location:  $l_0$
    - Action:  $s_{\text{Actuator}}$
    - Channel: left
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l_0$
  - New edge:
    - Source Location:  $l_0$
    - Action:  $f_{\text{Actuator}}$
    - Channel: right
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l_0$
  - New edge:
    - Source Location:  $l_0$
    - Action:  $f_{\text{Actuator}}$
    - Channel: left
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l_0$

**Input: timed process automaton Brake-by-Wire creation**

1. Name: Brake-by-Wire
2. Input Actions:  $\{signal1, signal2\}$
3. Output Actions:  $\{a\}$
4. Callees: Position
5. Clocks:  $\emptyset$
6. Channels:  $\{front, rear\}$
7. Locations:  $l0, l1$
8. Initial Location:  $l0$
9. Final Location:  $l1$
10. Invariant:  $l0 : true, l1 : true$
11. Edges:
  - New edge:
    - Source Location:  $l0$
    - Action:  $a$
    - Channel:  $*$
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l1$
  - New edge:
    - Source Location:  $l0$
    - Action:  $s_{Position}$
    - Channel:  $front$
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l0$
  - New edge:
    - Source Location:  $l0$
    - Action:  $s_{Position}$
    - Channel:  $rear$
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l0$
  - New edge:
    - Source Location:  $l0$
    - Action:  $f_{Position}$
    - Channel:  $front$
    - Clock Constraint:  $\emptyset$
    - Clock Resets:  $\emptyset$
    - Destination Location:  $l0$
  - New edge:
    - Source Location:  $l0$
    - Action:  $f_{Position}$
    - Channel:  $rear$
    - Clock Constraint:  $\emptyset$

- Clock Resets:  $\emptyset$
- Destination Location:  $l0$
- New edge:
  - Source Location:  $l0$
  - Action: **signal1**
  - Channel:  $*$
  - Clock Constraint:  $\emptyset$
  - Clock Resets:  $\emptyset$
  - Destination Location:  $l0$
- New edge:
  - Source Location:  $l0$
  - Action: **signal2**
  - Channel:  $*$
  - Clock Constraint:  $\emptyset$
  - Clock Resets:  $\emptyset$
  - Destination Location:  $l0$

**Output: timed process automaton *Actuator* display**

- *Name* = *Actuator*
- *Locations* =  $\{l0 : true, l1 : true\}$
- *initialLocation* =  $l0$
- *finalLocation* =  $l1$
- *Edges* =  $\{(l0, brake!, *, \emptyset, \emptyset, l1)\}$

**Output: timed process automaton *Position* display**

- *Name* = *Position*
- *Locations* =  $\{l0 : true, l1 : true\}$
- *initialLocation* =  $l0$
- *finalLocation* =  $l1$
- *Edges* =  $\{(l0, a!, *, \emptyset, \emptyset, l1), (l0, s_{Actuator}, right, \emptyset, \emptyset, l0), (l0, s_{Actuator}, left, \emptyset, \emptyset, l0), (l0, f_{Actuator}, right, \emptyset, \emptyset, l0), (l0, f_{Actuator}, left, \emptyset, \emptyset, l0)\}$

**Output: timed process automaton *Brake-by-Wire* display**

- *Name* = *Brake-by-Wire*
- *Locations* =  $\{l0 : true, l1 : true\}$
- *initialLocation* =  $l0$
- *finalLocation* =  $l1$
- *Edges* =  $\{(l0, a!, *, \emptyset, \emptyset, l1), (l0, s_{Position}, right, \emptyset, \emptyset, l0), (l0, s_{Position}, left, \emptyset, \emptyset, l0), (l0, f_{Position}, right, \emptyset, \emptyset, l0), (l0, f_{Position}, left, \emptyset, \emptyset, l0), (l0, signal1?, *, \emptyset, \emptyset, l0), (l0, signal2?, *, \emptyset, \emptyset, l0)\}$

### A.1 Output: a monolithic analysis model of timed process automaton **Actuator**

- Timed game analysis of  $\text{root}(P_0)$ 
  - $\text{root}(P_0)$  is
    - \*  $\text{Identifier} = P_0$
    - \*  $\text{tpa}(P_0) = \text{Actuator}$
    - \*  $\text{channel}(P_0) = \perp$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_0} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0$
    - \*  $\text{Edges} = \{(l_0, P_0.*\text{brake}!, \emptyset, \emptyset, \{x^{P_0}\}, l_1), (l_1, \perp.\text{f}_{\text{Actuator}}!, n = 0 \wedge x^{P_0} = 0, \emptyset, \emptyset, l_0^{P_0}), (l_1, P_0.*u?, n = 0 \wedge x^{P_0} > 0, \emptyset, \emptyset, \text{BAD})\}$

### A.2 Output: a monolithic analysis model of timed process automaton **Position**

- Timed game analysis of  $\text{root}(P_0) \parallel \text{standalone}(P_1) \parallel \text{standalone}(P_2)$ 
  - $\text{root}(P_0)$  is
    - \*  $\text{Identifier} = P_0$
    - \*  $\text{tpa}(P_0) = \text{Position}$
    - \*  $\text{channel}(P_0) = \perp$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_0} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0$
    - \*  $\text{Edges} = \{(l_0, P_0.*a!, \emptyset, \emptyset, \{x^{P_0}\}, l_1), (l_0, P_0.\text{right.s}_{\text{Actuator}}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_0.\text{left.s}_{\text{Actuator}}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_0.\text{right.f}_{\text{Actuator}}?, \emptyset, \{n--\}, \emptyset, l_0), (l_0, P_0.\text{left.f}_{\text{Actuator}}?, \emptyset, \{n--\}, \emptyset, l_0), (l_1, \perp.\text{f}_{\text{Position}}!, n = 0 \wedge x^{P_0} = 0, \emptyset, \emptyset, l_0^{P_0}), (l_1, P_0.*u?, n = 0 \wedge x^{P_0} > 0, \emptyset, \emptyset, \text{BAD})\}$
  - $\text{standalone}(P_1)$  is
    - \*  $\text{Identifier} = P_1$
    - \*  $\text{tpa}(P_1) = \text{Actuator}$
    - \*  $\text{channel}(P_1) = P_0.\text{right}$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_1} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0^{P_1}$
    - \*  $\text{Edges} = \{(l_0, P_1.*\text{brake}!, \emptyset, \emptyset, \{x^{P_1}\}, l_1), (l_1, P_0.\text{right.f}_{\text{Actuator}}!, n = 0 \wedge x^{P_1} = 0, \emptyset, \emptyset, l_0^{P_1}), (l_0^{P_1}, P_0.\text{right.s}_{\text{Actuator}}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_1.*u?, n = 0 \wedge x^{P_1} > 0, \emptyset, \emptyset, \text{BAD})\}$
  - $\text{standalone}(P_2)$  is
    - \*  $\text{Identifier} = P_2$
    - \*  $\text{tpa}(P_2) = \text{Actuator}$
    - \*  $\text{channel}(P_2) = P_0.\text{left}$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_2} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0^{P_2}$
    - \*  $\text{Edges} = \{(l_0, P_2.*\text{brake}!, \emptyset, \emptyset, \{x^{P_2}\}, l_1), (l_1, P_0.\text{left.f}_{\text{Actuator}}!, n = 0 \wedge x^{P_2} = 0, \emptyset, \emptyset, l_0^{P_2}), (l_0^{P_2}, P_0.\text{left.s}_{\text{Actuator}}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_2.*u?, n = 0 \wedge x^{P_2} > 0, \emptyset, \emptyset, \text{BAD})\}$



### A.3 Output: a monolithic analysis model of timed process automaton Brake-by-Wire

- Timed game analysis of  $\text{root}(P_0) \parallel \text{standalone}(P_1) \parallel \text{standalone}(P_2) \parallel \text{standalone}(P_3) \parallel \text{standalone}(P_4) \parallel \text{standalone}(P_5) \parallel \text{standalone}(P_6)$ 
  - $\text{root}(P_0)$  is
    - \*  $\text{Identifier} = P_0$
    - \*  $\text{tpa}(P_0) = \text{Brake-by-Wire}$
    - \*  $\text{channel}(P_0) = \perp$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_0} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0$
    - \*  $\text{Edges} = \{(l_0, P_0.*a!, \emptyset, \emptyset, \{x^{P_0}\}, l_1), (l_0, P_0.\text{front.sPosition}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_0.\text{rear.sPosition}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_0.\text{front.fPosition}?, \emptyset, \{n--\}, \emptyset, l_0), (l_0, P_0.\text{rear.fPosition}?, \emptyset, \{n--\}, \emptyset, l_0), (l_1, \perp.\text{fBrake-by-Wire}!, n = 0 \wedge x^{P_0} = 0, \emptyset, \emptyset, l_0^{P_0}), (l_1, P_0.*u?, n = 0 \wedge x^{P_0} > 0, \emptyset, \emptyset, \text{BAD}), (l_0, P_0.*\text{signal1}?, \emptyset, \emptyset, \emptyset, l_0), (l_0, P_0.*\text{signal2}?, \emptyset, \emptyset, \emptyset, l_0)\}$
  - $\text{standalone}(P_1)$  is
    - \*  $\text{Identifier} = P_1$
    - \*  $\text{tpa}(P_1) = \text{Position}$
    - \*  $\text{channel}(P_1) = P_0.\text{front}$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_1} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0^{P_1}$
    - \*  $\text{Edges} = \{(l_0, P_1.*a!, \emptyset, \emptyset, \{x^{P_1}\}, l_1), (l_0, P_1.\text{right.sActuator}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_1.\text{left.sActuator}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_1.\text{right.fActuator}?, \emptyset, \{n--\}, \emptyset, l_0), (l_0, P_1.\text{left.fActuator}?, \emptyset, \{n--\}, \emptyset, l_0), (l_1, P_0.\text{front.fPosition}!, n = 0 \wedge x^{P_1} = 0, \emptyset, \emptyset, l_0^{P_1}), (l_0^{P_1}, P_0.\text{front.sPosition}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_1.*u?, n = 0 \wedge x^{P_1} > 0, \emptyset, \emptyset, \text{BAD})\}$
  - $\text{standalone}(P_2)$  is
    - \*  $\text{Identifier} = P_2$
    - \*  $\text{tpa}(P_2) = \text{Position item channel}(P_2) = P_0.\text{rear}$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_2} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0^{P_2}$
    - \*  $\text{Edges} = \{(l_0, P_2.*a!, \emptyset, \emptyset, \{x^{P_2}\}, l_1), (l_0, P_2.\text{right.sActuator}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_2.\text{left.sActuator}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_2.\text{right.fActuator}?, \emptyset, \{n--\}, \emptyset, l_0), (l_0, P_2.\text{left.fActuator}?, \emptyset, \{n--\}, \emptyset, l_0), (l_1, P_0.\text{rear.fPosition}!, n = 0 \wedge x^{P_2} = 0, \emptyset, \emptyset, l_0^{P_2}), (l_0^{P_2}, P_0.\text{rear.sPosition}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_2.*u?, n = 0 \wedge x^{P_2} > 0, \emptyset, \emptyset, \text{BAD})\}$
  - $\text{standalone}(P_3)$  is
    - \*  $\text{Identifier} = P_3$
    - \*  $\text{tpa}(P_3) = \text{Actuator}$
    - \*  $\text{channel}(P_3) = P_1.\text{right}$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_3} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0^{P_3}$
    - \*  $\text{Edges} = \{(l_0, P_3.*\text{brake}!, \emptyset, \emptyset, \{x^{P_3}\}, l_1), (l_1, P_1.\text{right.fActuator}!, n = 0 \wedge x^{P_3} = 0, \emptyset, \emptyset, l_0^{P_3}), (l_0^{P_3}, P_1.\text{right.sActuator}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_3.*u?, n = 0 \wedge x^{P_3} > 0, \emptyset, \emptyset, \text{BAD})\}$

- $\text{standalone}(P_4)$  is
  - \*  $\text{Identifier} = P_4$
  - \*  $\text{tpa}(P_4) = \text{Actuator}$
  - \*  $\text{channel}(P_4) = P_1.\text{left}$
  - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_4} : \text{true}, \text{BAD} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0^{P_4}$
  - \*  $\text{Edges} = \{(l_0, P_4.*\text{brake!}, \emptyset, \emptyset, \{x^{P_4}\}, l_1), (l_1, P_1.\text{left.f}_{\text{Actuator}}!, n = 0 \wedge x^{P_4} = 0, \emptyset, \emptyset, l_0^{P_4}), (l_0^{P_4}, P_1.\text{left.s}_{\text{Actuator}}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_4.*.u?, n = 0 \wedge x^{P_4} > 0, \emptyset, \emptyset, \text{BAD})\}$
- $\text{standalone}(P_5)$  is
  - \*  $\text{Identifier} = P_5$
  - \*  $\text{tpa}(P_5) = \text{Actuator}$
  - \*  $\text{channel}(P_5) = P_2.\text{right}$
  - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_5} : \text{true}, \text{BAD} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0^{P_5}$
  - \*  $\text{Edges} = \{(l_0, P_5.*\text{brake!}, \emptyset, \emptyset, \{x^{P_5}\}, l_1), (l_1, P_2.\text{right.f}_{\text{Actuator}}!, n = 0 \wedge x^{P_5} = 0, \emptyset, \emptyset, l_0^{P_5}), (l_0^{P_5}, P_2.\text{right.s}_{\text{Actuator}}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_5.*.u?, n = 0 \wedge x^{P_5} > 0, \emptyset, \emptyset, \text{BAD})\}$
- $\text{standalone}(P_6)$  is
  - \*  $\text{Identifier} = P_6$
  - \*  $\text{tpa}(P_6) = \text{Actuator}$
  - \*  $\text{channel}(P_6) = P_2.\text{left}$
  - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_6} : \text{true}, \text{BAD} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0^{P_6}$
  - \*  $\text{Edges} = \{(l_0, P_6.*\text{brake!}, \emptyset, \emptyset, \{x^{P_6}\}, l_1), (l_1, P_2.\text{left.f}_{\text{Actuator}}!, n = 0 \wedge x^{P_6} = 0, \emptyset, \emptyset, l_0^{P_6}), (l_0^{P_6}, P_2.\text{left.s}_{\text{Actuator}}?, \emptyset, \emptyset, \emptyset, l_0), (l_1, P_6.*.u?, n = 0 \wedge x^{P_6} > 0, \emptyset, \emptyset, \text{BAD})\}$

#### A.4 Output: a compositional analysis model of timed process automaton **Actuator**

- Timed game analysis of  $\text{root}(P_0)$ 
  - $\text{root}(P_0)$  is
    - \*  $\text{Identifier} = P_0$
    - \*  $\text{tpa}(P_0) = \text{Actuator}$
    - \*  $\text{channel}(P_0) = \perp$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_0} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0$
    - \*  $\text{Edges} = \{(l_0, P_0.*\text{brake!}, \emptyset, \emptyset, \{x^{P_0}\}, l_1), (l_1, \perp.\text{f}_{\text{Actuator}}!, n = 0 \wedge x^{P_0} = 0, \emptyset, \emptyset, l_0^{P_0}), (l_1, P_0.*.u?, n = 0 \wedge x^{P_0} > 0, \emptyset, \emptyset, \text{BAD})\}$

#### A.5 Output: a compositional analysis model of timed process automaton **Position**

- Timed game analysis of  $\text{root}(P_0) \parallel \text{duration}(P_1) \parallel \text{duration}(P_2)$

- $\text{root}(P_0)$  is
  - \*  $\text{Identifier} = P_0$
  - \*  $\text{tpa}(P_0) = \text{Position}$
  - \*  $\text{channel}(P_0) = \perp$
  - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_0} : \text{true}, \text{BAD} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0$
  - \*  $\text{Edges} = \{(l_0, P_0.*a!, \emptyset, \emptyset, \{x^{P_0}\}, l_1), (l_0, P_0.\text{right.sActuator}!, \emptyset, \{n++\}, \emptyset, l_0),$   
 $(l_0, P_0.\text{left.sActuator}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_0.\text{right.fActuator}?, \emptyset, \{n--\}, \emptyset, l_0),$   
 $(l_0, P_0.\text{left.fActuator}?, \emptyset, \{n--\}, \emptyset, l_0), (l_1, \perp.\text{fPosition}!, n = 0 \wedge x^{P_0} = 0, \emptyset, \emptyset, l_0^{P_0}), (l_1, P_0.*$   
 $.u?, n = 0 \wedge x^{P_0} > 0, \emptyset, \emptyset, \text{BAD})\}$
- $\text{duration}(P_1)$  is
  - \*  $\text{Identifier} = P_1$
  - \*  $\text{tpa}(P_1) = \text{Actuator}$
  - \*  $\text{channel}(P_1) = P_0.\text{right}$
  - \*  $\text{The WCET (input from the game analyzer)} = \text{Unknown}$
  - \*  $\text{Locations} = \{l_0^{P_1} : \text{true}, l_1^{P_1} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0^{P_1}$
  - \*  $\text{Edges} = \{(l_1^{P_1}, P_0.\text{right.fActuator}!, \emptyset, \emptyset, \emptyset, l_0^{P_1}), (l_0^{P_1}, P_0.\text{right.sActuator}?, \emptyset, \emptyset, \emptyset, l_1^{P_1})\}$
- $\text{duration}(P_2)$  is
  - \*  $\text{Identifier} = P_2$
  - \*  $\text{tpa}(P_2) = \text{Actuator}$
  - \*  $\text{channel}(P_2) = P_0.\text{left}$
  - \*  $\text{The WCET (input from the game analyzer)} = \text{Unknown}$
  - \*  $\text{Locations} = \{l_0^{P_2} : \text{true}, l_1^{P_2} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0^{P_2}$
  - \*  $\text{Edges} = \{(l_1^{P_2}, P_0.\text{left.fActuator}!, \emptyset, \emptyset, \emptyset, l_0^{P_2}), (l_0^{P_2}, P_0.\text{left.sActuator}?, \emptyset, \emptyset, \emptyset, l_1^{P_2})\}$

## A.6 Output: a compositional analysis model of timed process automaton Brake-by-Wire

- Timed game analysis of  $\text{root}(P_0) || \text{duration}(P_1) || \text{duration}(P_2)$ 
  - $\text{root}(P_0)$  is
    - \*  $\text{Identifier} = P_0$
    - \*  $\text{tpa}(P_0) = \text{Brake-by-Wire}$
    - \*  $\text{channel}(P_0) = \perp$
    - \*  $\text{Locations} = \{l_0 : \text{true}, l_1 : \text{true}, l_0^{P_0} : \text{true}, \text{BAD} : \text{true}\}$
    - \*  $\text{initialLocation} = l_0$
    - \*  $\text{Edges} = \{(l_0, P_0.*a!, \emptyset, \emptyset, \{x^{P_0}\}, l_1), (l_0, P_0.\text{front.sPosition}!, \emptyset, \{n++\}, \emptyset, l_0),$   
 $(l_0, P_0.\text{rear.sPosition}!, \emptyset, \{n++\}, \emptyset, l_0), (l_0, P_0.\text{front.fPosition}?, \emptyset, \{n--\}, \emptyset, l_0),$   
 $(l_0, P_0.\text{rear.fPosition}?, \emptyset, \{n--\}, \emptyset, l_0), (l_1, \perp.\text{fBrake-by-Wire}!, n = 0 \wedge x^{P_0} =$   
 $0, \emptyset, \emptyset, l_0^{P_0}), (l_1, P_0.*u?, n = 0 \wedge x^{P_0} > 0, \emptyset, \emptyset, \text{BAD}), (l_0, P_0.*\text{signal1}?, \emptyset, \emptyset, \emptyset, l_0),$   
 $(l_0, P_0.*\text{signal2}?, \emptyset, \emptyset, \emptyset, l_0)\}$
  - $\text{duration}(P_1)$  is
    - \*  $\text{Identifier} = P_1$
    - \*  $\text{tpa}(P_1) = \text{Position}$

- \*  $\text{channel}(P_1) = P_0.\text{front}$
- \* *The WCET (input from the game analyzer) = Unknown*
- \*  $\text{Locations} = \{l_0^{P_1} : \text{true}, l_1^{P_1} : \text{true}\}$
- \*  $\text{initialLocation} = l_0^{P_1}$
- \*  $\text{Edges} = \{(l_1^{P_1}, P_0.\text{front}.\text{fPosition}!, \emptyset, \emptyset, \emptyset, l_0^{P_1}), (l_0^{P_1}, P_0.\text{front}.\text{sPosition}?, \emptyset, \emptyset, \emptyset, l_1^{P_1})\}$
- $\text{duration}(P_2)$  is
  - \*  $\text{Identifier} = P_2$
  - \*  $\text{tpa}(P_2) = \text{Position}$
  - \*  $\text{channel}(P_2) = P_0.\text{rear}$
  - \* *The WCET (input from the game analyzer) = Unknown*
  - \*  $\text{Locations} = \{l_0^{P_2} : \text{true}, l_1^{P_2} : \text{true}\}$
  - \*  $\text{initialLocation} = l_0^{P_2}$
  - \*  $\text{Edges} = \{(l_1^{P_2}, P_0.\text{rear}.\text{fPosition}!, \emptyset, \emptyset, \emptyset, l_0^{P_2}), (l_0^{P_2}, P_0.\text{rear}.\text{sPosition}?, \emptyset, \emptyset, \emptyset, l_1^{P_2})\}$