CISC 203 - Assignment 1 (Fall 2019)
Due: Thursday September 26 **by 2:30 PM**
(in the locked CISC 203 drop-off box on Goodwin 2nd floor)

One bonus mark for neatly written student information: Papers that have your name, student number and other information written <u>exactly</u> as requested in the regulations (found at the end), will receive one bonus mark.

1. (3 marks) Let A and B be sets. Prove

$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

Recall that $\overline{A}$ stands for the complement of the set A.

Note: In order to prove equality of sets, you need to prove inclusion in both directions. If you wish, you can illustrate the argument using a Venn diagram but a diagram by itself is <u>not</u> sufficient. You are expected to give a logical argument that proves both inclusions.

2. Consider relations $R \subseteq A \times B$ and $S_1, S_2 \subseteq B \times C$ (where A, B, C are sets).

   (a) (2 marks) Prove that $R \odot (S_1 \cap S_2) \subseteq (R \odot S_1) \cap (R \odot S_2)$.

   (b) (2 marks) Give a concrete example of relations $R \subseteq A \times B$ and $S_1, S_2 \subseteq B \times C$ such that

   $$R \odot (S_1 \cap S_2) \neq (R \odot S_1) \cap (R \odot S_2)$$

   (thus proving that the inclusion in part (a) cannot be replaced by equality).

   Above $R \odot S$ denotes the composition of relations R and S (as defined in Definition 3.2 of the first part of the notes).

   Hint for (b): It may be useful to select $S_1, S_2 \neq \emptyset$ such that $S_1 \cap S_2 = \emptyset$.

3. (3 marks) For each case answer "true" or "false". (In this question it is not required to explain the answers.)

   - (a) $5n^2 + n + 7 = O(n^2)$
   - (b) $n^2 = o(5n^2 + n + 7)$
   - (c) $n^2 = o(n^2 \cdot \sqrt{n})$
   - (d) $n^2 = O(n^2 \cdot \sqrt{n})$
   - (e) $\log n = o(\sqrt{n})$
   - (f) $\sqrt{n} = O(\log n)$

4. (4 marks) A decimal digit is one of the characters 0, 1, 2, ..., 9. A string is an ordered sequence of characters. Examples of strings of four decimal digits include 1234, 0001, 5567.

   How many strings of four decimal digits

   (a) do not contain the same digit four times?
   (b) do not contain the same digit twice?
   (c) end with an even digit?
   (d) have exactly 3 digits that are 8's?

   <u>Justify</u> your answers: in each case you need to explain why the number is as claimed.

5. (2 marks) Show that if f is a function from A to B, where A and B are finite sets and $\ell = \lceil \frac{|A|}{|B|} \rceil$, then there are at least ℓ elements of A mapped to the same element of B. That is, show that there are ℓ distinct elements $a_1, \ldots, a_\ell \in A$ such that $f(a_1) = \ldots = f(a_\ell)$.

   Hint. Here some result from class (the notes on combinatorics) may be useful.

6. (4 marks) Consider a function $f : A \to B$. Prove:

   f has a right inverse if and only if f is onto.

   Recall that "right inverse of a function" is defined in Definition 3.4 of the first set of notes.

- The assignment should be put into the locked CISC 203 drop-off box on the 2nd floor of Goodwin hall by the due date. The assignments <u>must be submitted in hardcopy</u>. Assignments sent by email are not accepted.

- If the submission consists of more than one page, the pages must be <u>stapled</u> together.

- *Note:* You are asked to write your solutions using non-erasable pen (or to type the solutions). Solutions written in pencil or erasable ink will be marked, but they will not be considered for remarking after the assignments are returned.