

Queen’s University, Faculty of Arts and Science, School of Computing
CISC-203 Practice Exam, November 2019 (Instructor: Kai Salomaa)

INSTRUCTIONS

- **Aids allowed:** You may bring in one 8.5 × 11 inch sheet of paper containing notes, and use it during the exam. The sheet can be written/printed on both sides.
- This examination is **THREE HOURS** in length. Answer all 10 questions.
- **Answer each question in the space provided** (on the question paper). There are two extra pages at the end of the exam, if more space is needed. **Please write legibly.**
- *Note:* In questions dealing with counting or combinatorics it is not expected that you should compute large numerical values: it is fine to give the final answer in a form like “ $\frac{56!}{29!}$ ” or “ $\binom{40}{21}$ ” as long as you clearly explain how you arrived at the answer.

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

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STUDENT NUMBER:

One digit in each square, please!

STUDENT NUMBER (written in words):

Problem 1	/X ₁
Problem 2	/X ₂
Problem 3	/X ₃
Problem 4	/X ₄
Problem 5	/X ₅
Problem 6	/X ₆
Problem 7	/X ₇
Problem 8	/X ₈
Problem 9	/X ₉
Problem 10	/X ₁₀
Total	/50

1. Consider permutations (with no repetition) of the sequence of nine letters ABCDEFGH. Answer the following questions and justify your answers.

(i) How many permutations contain the string FAD ?

(ii) How many permutations contain the string BADEF ?

(iii) How many permutations contain both the strings ABC and FGH ?

(iv) How many permutations contain both the strings CE and DE ?

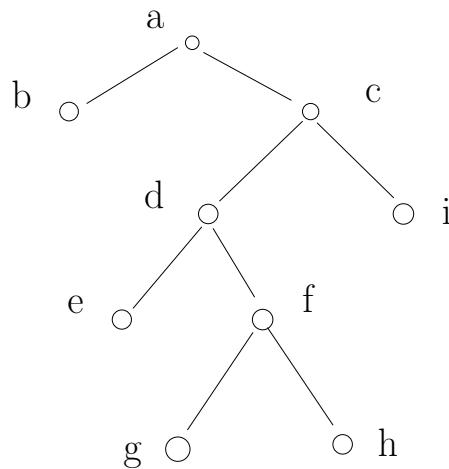


Figure 1: Tree T_1

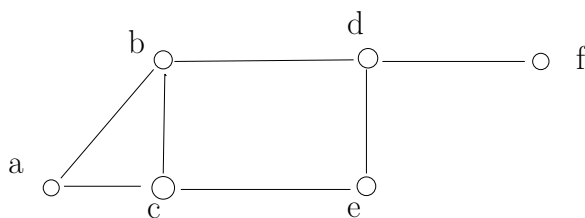
2. Consider the rooted ordered binary tree T_1 given in Figure 1

(i) In which order does the pre-order traversal visit the vertices of T_1 ?

(ii) In which order does the post-order traversal visit the vertices of T_1 ?

(iii) In which order does the in-order traversal visit the vertices of T_1 ?

(iv) In which order does the level-order traversal visit the vertices of T_1 ?

Figure 2: Graph G

3. Consider the graph G given in Figure 2.

(i) Determine whether G has a **Hamiltonian cycle** and construct a Hamiltonian cycle if one exists. If G does not have a Hamiltonian cycle give a rigorous argument to show why no Hamiltonian cycle exists.

(ii) Determine whether G has a **Hamiltonian path** and construct a Hamiltonian path if one exists. If G does not have a Hamiltonian path give a rigorous argument to show why no Hamiltonian path exists.

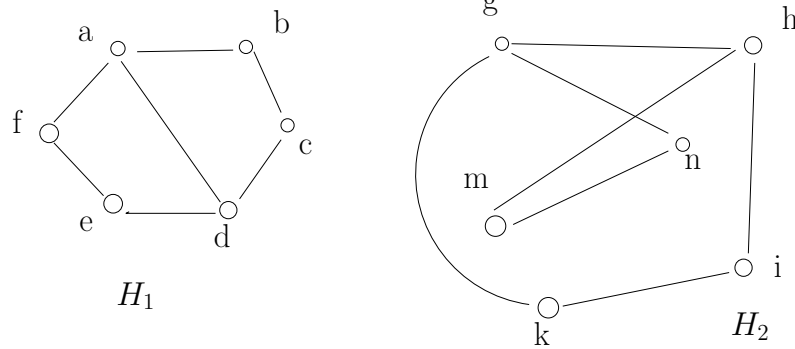


Figure 3: Graphs H_1 and H_2

4. (i) Give adjacency matrices for the graphs H_1 and H_2 given in Figure 3. In the matrices remember to clearly indicate what is the ordering of rows and columns.
- (ii) Are the graphs H_1 and H_2 given in Figure 3 isomorphic? Either give an explicit isomorphism between H_1 and H_2 or give a rigorous argument showing that an isomorphism does not exist.

5. Consider a function $f : A \rightarrow B$. Prove:

f has a left inverse if and only if f is one-to-one.

Recall that $h : B \rightarrow A$ is a left inverse of function f if $h \circ f = 1_A$.

Note: In order to prove an “iff” statement you need to prove the implication in both directions.

7. Tim Horton's at Bioscience has 4 different varieties of cookies.

In how many ways can 12 cookies be chosen? When choosing a number of cookies only the variety of the cookie matters. Also the order in which the selections are made does not matter. Justify your answer.

8. Roll two fair dice and consider the following events:

Event A: the numbers on the two dice sum up to 8

Event B: the numbers on the two dice are both even

Calculate the conditional probability $P(A | B)$ and show your work.

9. Using the characteristic root method find a solution for the recurrence

$$a_n = 3a_{n-1} + 4a_{n-2}$$

with initial values $a_0 = 3$ and $a_1 = 6$. That is, give a closed form expression for a_n as a function of n . This includes finding the values of the associated constants. Please remember to show your work.

10. Recall that K_n is the complete graph with n vertices.

(i) Show that K_4 is planar.

(ii) What is the chromatic number of K_7 ? Justify your answer.

(iii) Show that K_5 is nonplanar. You can assume known Euler's formula for planar graphs.

Do all 10 questions. Student#:

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1st extra page.

Do all 10 questions. Student#:

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2nd extra page.