QUEEN'S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF COMPUTING AND INFORMATION SCIENCE

CISC-204*
Logic for Computer Scientists

TEST 1
February 3, 2006

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Please write your answer to each question only in the box marked Answer.
No questions will be answered by the instructors during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.

NAME: ___________________________ SECTION:___________

STUDENT NUMBER: ________________

FOR INSTRUCTOR'S USE ONLY

Question 1: _____ / 10

Question 2: _____ / 10

Question 3: _____ / 10

Question 4: _____ / 10

TOTAL: _______ / 40
Question 1: [10 marks]

Prove the validity of the sequent:

\[ p \rightarrow (q \lor \neg r), \neg q, r \vdash \neg p \]

Justify each step of your proof. A copy of the basic rules for natural deduction is attached as the last page of the quiz.

Answer:
Question 2: [10 marks]

Using mathematical induction, prove the following property:

\[ n < 3^n - 1 \]

for all positive integers \( n \)

Answer:
Question 3: [10 marks]

For the formula

\[(\neg q \lor r) \rightarrow (r \rightarrow p)\]

Answer:

a) Put the formulae in conjunctive normal form (CNF) using a truth table.

b) Transform the formula into conjunctive normal form (CNF) using the algorithm given in class. Useful equivalences are listed on the attached page at the end of the quiz.
Question 4: [10 marks]

Is the following Horn formula satisfiable? ____.  
Apply the Horn mark up algorithm to justify your answer. If it is satisfiable, provide an assignment of truth values that satisfies the formula.

\[(p \land q \rightarrow \bot) \land (T \rightarrow q) \land (q \land r \rightarrow T) \land (p \land s \rightarrow r) \land (q \rightarrow r)\]

Answer: