CISC-204*
Test #3
March 10, 2005

Dr. Robin Dawes, Dr. Janice Glasgow

Student Number (Required) ________________________

Name (Optional)__________________________________

Section ___________________ (Dawes: A, Glasgow: B)

This is a closed book test. You may not refer to any resources other than the information sheets stapled to the back of the test. You may remove the information sheets.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned. Please write your answers in the boxes provided.

The test will be marked out of 40.

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TOTAL /40
QUESTION 1. 8 MARKS

Prove the validity of the following sequent in predicate logic

$$\exists x \ P(x) \vdash \neg (\forall x \ (\neg \ P(x)))$$
QUESTION 2: 10 MARKS

Translate the following argument into predicate logic, using the given predicates, and give a formal proof of validity.

If there are any letters to the editor, then all students are intellectuals.
If there are no letters to the editor, then there are no readers.
Therefore, if there are any readers and any students, then there is at least one intellectual.

Predicates:
- L(x) : x is a letter to the editor
- S(x) : x is a student
- I(x) : x is an intellectual
- R(x) : x is a reader
QUESTION 3: 10 MARKS

Let \( \phi \) be the formula
\[
\forall x \left( \neg P(x,x) \land \exists y \left( P(x,y) \lor P(y,x) \right) \right)
\]

(a) Find a model \( \mathcal{M} \) that satisfies \( \phi \)

(b) Find a model \( \mathcal{M}' \) that satisfies \( \neg \phi \)  
   (Hint: you may wish to simplify \( \neg \phi \). If so, show your work.)
QUESTION 4: 12 MARKS

For each of the following sets of formulae, show that the set is consistent or prove that it is not consistent.

a) 1. $\forall x \forall y \ ((P(x) \land P(y)) \rightarrow P(f(x,y)))$
2. $\neg \exists x \ (f(x,x) = x)$
3. $\exists x \ P(x)$

b) 1. $\forall x \ (P(x) \rightarrow Q(x))$
2. $\forall x \ P(x)$
3. $\exists x \ \neg Q(x)$