QUEEN’S UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF COMPUTING AND INFORMATION SCIENCE

CISC-204*
Logic for Computer Scientists

TEST 4
March 24, 2006

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Please write your answer to each question only in the box marked Answer.
No questions will be answered by the instructors during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question,
make a reasonable assumption and write this at the beginning of your answer.

NAME: ___________________________ SECTION:___________

STUDENT NUMBER: ______________

FOR INSTRUCTOR’S USE ONLY

Question 1: _____ / 10

Question 2: _____ / 10

Question 3: _____ / 10

Question 4: _____ / 10

TOTAL: ______ / 40
Question 1: [10 marks]

Using the definition attached at the end of the quiz, prove the following logical equivalence:

\[ \phi W \psi \equiv \phi U \psi \lor G \phi \]

Answer:
Question 2: [10 marks]

Consider the model $\mathcal{M}$ described below. For each of the formulas $\phi$ listed, find a path from the initial state $s_0$ that satisfies $\phi$ and determine whether $\mathcal{M}, s_0 \models \phi$. Justify your answer.

\begin{center}
\includegraphics[width=0.3\textwidth]{model.png}
\end{center}

Answer:

(a) $p \rightarrow XX\neg q$

(b) $G(p \rightarrow Xq)$

(c) $FGq$
Question 3: [10 marks]

Assume you have a system with three processes, where each process can be in one of two states, off or on, and there are transition relations on \(\rightarrow\) off and off \(\rightarrow\) on for each process. For any state transition, exactly one process can change its state.

**Answer:**

Define the atomic propositions needed to model this system.

Express each of the following properties in linear-time temporal logic using the propositions defined above.
- All processes are off in the initial state for the model.

- It is not possible for all three processes to be in their on state at any given time.

- Whenever process 1 is in its on state, then in the next state process 2 must be off.
Question 4: [10 marks]

Using a transition diagram, construct a model for the system that satisfies the properties expressed in Question 3.

Answer:
Definition: Let $\mathcal{M} = \{S, \rightarrow, L\}$ be a model and $\pi = s_1 \rightarrow ...$ be a path in $\mathcal{M}$. Whether path $\pi$ satisfies an LTL formula is defined by the satisfacttion $\models$ relation as follows:

1. $\pi \models \top$

2. $\pi \not\models \bot$

3. $\pi \models p$ iff $p \in L(s_1)$

4. $\pi \models \neg \phi$ iff $\pi \not\models \phi$

5. $\pi \models \phi \land \psi$ iff $\pi \models \phi$ and $\pi \models \psi$

6. $\pi \models \phi \lor \psi$ iff $\pi \models \phi$ or $\pi \models \psi$

7. $\pi \models \phi \rightarrow \psi$ iff $\pi \models \psi$ whenever $\pi \models \phi$

8. $\pi \models X\phi$ iff $\pi^2 \models \phi$

9. $\pi \models G\phi$ iff $\pi^i \models \phi$ for all $i \geq 1$

10. $\pi \models F\phi$ iff $\pi^i \models \phi$ for some $i \geq 1$

11. $\pi \models \phi U \psi$ iff there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, ..., i - 1$ we have $\pi^j \models \phi$

12. $\pi \models \phi W \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, ..., i - 1$ we have $\pi^j \models \phi$; or for all $k \geq 1$ we have $\pi^k \models \phi$

13. $\pi \models \phi R \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \phi$ and for all $j = 1, ..., i$ we have $\pi^j \models \psi$; or for all $k \geq 1$ we have $\pi^k \models \psi$. 