CISC 204 Class 7

Basic Semantics of Propositional Logic

Text Correspondence: pp. 31–40

Main Concepts:

- *WFF definition*
- *Valuation of an atom: assignment to a truth value*
- *Model of a formula: systematic valuation of each atom*
- *Truth table: tabular form of models*

The *semantics* of propositional logic are a way to assign "meaning" to propositions. To begin, we will refine our understanding of a well formed formula by defining a WFF more carefully.

7.1 Well Formed Formula, or WFF

Definition: Well-formed formula

The *well-formed formulas* of propositional logic are those which we obtain by using the construction rules below, and only those.

- atomic formulas: Every propositional atom $p, q, r, \ldots$ is a well-formed formula.
- complex formulas: if ϕ and ψ are well-formed formulas, then
  - $(\neg\,\phi)$ is a well-formed formula
  - $(\phi \wedge \psi)$ is a well-formed formula
  - $(\phi \vee \psi)$ is a well-formed formula
  - $(\phi \rightarrow \psi)$ is a well-formed formula

Example: is this a well-formed formula?

$$((\neg\, p) \wedge q) \rightarrow (p \wedge (q \vee (\neg\, r)))$$

A well-formed formula can be expressed as a *parse tree* where the atomic formula are the leaves of the tree and the logical operators are the root and intermediate nodes.

$$((\neg\, p) \wedge q) \rightarrow (p \wedge (q \vee (\neg\, r)))$$

H&R, p. 40

© R E Ellis 2022

7.2 Semantics of Propositional Logic

The semantics of proposition logic give us a *valuation*, or a *model*, of any formula in natural deduction.

Definition: Truth value

The set of *truth values* contains two elements T and F, denoting true and false.

Definition: Valuation

A *valuation* or *model* of a formula ϕ is an assignment of each propositional atom in ϕ to a truth value.

A common means of writing the semantics of propositional logic is to use a *truth table*, which is a model written in tabular form. The truth tables for the four basic operators are:

ϕ	ψ	$\phi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

ϕ	ψ	$\phi \rightarrow \psi$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	$\neg\phi$
T	F
F	T

© R E Ellis 2022

We can construct a truth table for any logical formula. For example:

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \lor \neg p$	$(p \to \neg q) \to (q \lor \neg p)$
T	**T**	**F**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**	**T**

It is important to understand that, for the purposes of this course, a truth table is not a proof. In natural deduction, there is a clear linkage between a truth table and a proof but these are not the same: a truth table is a semantics concept; a valid proof is the result of applying rules of deduction to the premises of a sequent, producing the conclusion of the sequent.

A truth table may be useful as a guide. For example, a truth table might be used to determine whether or not to proceed with a proof. But a formula should not be taken to be a theorem until a valid [constructive] proof has been produced.

© R E Ellis 2022