

## Reference: Proof Rules in Natural Deduction

Type	<i>introduction</i>	<i>elimination</i>
$\wedge$	$\frac{\phi \quad \psi}{\phi \wedge \psi} \wedge i$	$\frac{\phi \wedge \psi}{\phi} \wedge e_1 \quad \frac{\phi \wedge \psi}{\psi} \wedge e_2$
$\vee$	$\frac{\phi}{\phi \vee \psi} \vee i_1 \quad \frac{\psi}{\phi \vee \psi} \vee i_2$	$\frac{\phi \vee \psi \quad \begin{array}{ c } \hline \phi \\ \vdots \\ \chi \\ \hline \end{array} \quad \begin{array}{ c } \hline \psi \\ \vdots \\ \chi \\ \hline \end{array}}{\chi} \vee e$
$\rightarrow$	$\frac{\begin{array}{ c } \hline \phi \\ \vdots \\ \psi \\ \hline \end{array}}{\phi \rightarrow \psi} \rightarrow i$	$\frac{\phi \quad \phi \rightarrow \psi}{\psi} \rightarrow e$
$\neg$	$\frac{\begin{array}{ c } \hline \phi \\ \vdots \\ \perp \\ \hline \end{array}}{\neg \phi} \neg i$	$\frac{\phi \quad \neg \phi}{\perp} \neg e$
$\perp$	(none)	$\frac{\perp}{\phi} \perp e$
$\neg\neg$	$\frac{\phi}{\neg\neg\phi} \neg\neg i$	$\frac{\phi \quad \neg\neg\phi}{\phi} \neg\neg e$
Derived rules:	$\frac{\begin{array}{ c } \hline \neg\phi \\ \vdots \\ \perp \\ \hline \end{array}}{\phi} \text{PBC}$	$\frac{\phi \rightarrow \psi \quad \neg\psi}{\neg\phi} \text{MT}$
Other rules:	$\frac{}{\phi \vee \neg\phi} \text{LEM}$	$\frac{\phi}{\phi} \text{copy}$

## Rules of Deduction in Predicate Logic

The basic rules in predicate logic, with the textbook requirement of “free for  $x$ ”:

	<i>introduction</i>	<i>elimination</i>
$\forall x$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math>x_0</math>  <math>\vdots</math>  <math>\phi[x_0/x]</math> </div> <hr style="width: 80%; margin: 5px auto;"/> $\forall x \phi$ <div style="text-align: right; margin-top: -10px;"><math>\forall x i</math></div>	$\forall x \phi$ <hr style="width: 80%; margin: 5px auto;"/> $\phi[t/x]$ <div style="text-align: right; margin-top: -10px;"><math>\forall x e</math></div>
$\exists x$	$\phi[t/x]$ <hr style="width: 80%; margin: 5px auto;"/> $\exists x \phi$ <div style="text-align: right; margin-top: -10px;"><math>\exists x i</math></div>	$\exists x \phi$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;"> <math>x_0</math>    <math>\phi[x_0/x]</math>  <math>\vdots</math>  <math>\chi</math> </div> <hr style="width: 80%; margin: 5px auto;"/> $\chi$ <div style="text-align: right; margin-top: -10px;"><math>\exists x e</math></div>