CISC 204  Class 1

Course Introduction
Text Correspondence: pp. 1–5

This course is an introduction to formal logic, which is the study of abstract reasoning and deduction. We will use symbols to represent elementary “things” that are either true or false, and study ways of proving that complex statements are true or false (within a given logical system).

Logic from an historical perspective:

- **800 BC**: Greeks rediscovered writing.
- **600 BC**: Mathematics as a deductive science was introduced by Thales of Miletus.
- **500 BC**: Pythagoras introduced natural numbers.
- **350 BC**: Aristotle introduced logic as a way of determining whether reasoning was correct. Introduced the *syllogism*:

  All birds can fly
  Tweety is a bird
  Therefore Tweety can fly

Example of a logical argument:

If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late. *Therefore*, there were taxis at the station.

Form of this logical argument:

\[ p: \text{The train arrives late} \]
\[ q: \text{There are taxis at the station} \]
\[ r: \text{John is late for his meeting} \]

If \( p \) and not \( q \) then \( r \). Not \( r \). \( p \).
Therefore, \( q \)
Argument that is not logical:

If a course is fun to learn then you will do well on the exam. Jane did well on the CISC 204 exam, therefore it was fun to learn.

Propositional Logic

- proposition: a declarative sentence that can take on a value of true or false.
  - The sum of 2 and 2 is equal to 5.
  - Today is Thursday.
  - Every rational number greater than 2 can be expressed as the product of prime numbers.
  - The moon is made of green cheese.

Syntax for Propositional Logic

- Symbols $p, q, r, \ldots$ are used to denote *atomic propositions*. For example,
  
  $p$: It is raining
  $q$: It is winter
  $r$: It is cold

- We can form *complex sentences (propositions)* using logical connectives:

  $\neg p$ denotes not $p$, or it is not the case that $p$ is true.
  $p \lor q$ denotes that $p$ is true or $q$ is true.
  $p \land q$ denotes that $p$ is true and $q$ is true.
  $p \rightarrow q$ denotes that if $p$ is true then $q$ is true.

How would you express:

If it is cold and not raining outside then it must be winter.
Bracketing Conventions:
\( \neg \) binds more tightly than \( \land \) or \( \lor \), and the latter two more tightly than \( \rightarrow \). Implication \( \rightarrow \) is right-associative:

\[ p \rightarrow q \rightarrow r \text{ denotes } p \rightarrow (q \rightarrow r) \]

When in doubt, use brackets!!

Take home problem. Express the following declarative sentence in propositional logic. Define your atomic symbols.

If you study hard and do all of the assignments then you will get a good mark in the class, but if you do not study or do not do all of the assignments then you will not get a good mark in the class.