The take-home problem from the previous class was to express the following declarative sentence in propositional logic. You were to define the atomic symbols.

If you study hard and do all of the assignments then you will get a good mark in the class, but if you do not study or do not do all of the assignments then you will not get a good mark in the class.

An example solution is:

\[ p: \text{ You study hard.} \]
\[ q: \text{ You do all of the assignments.} \]
\[ r: \text{ You get a good mark in the class.} \]

How would you represent the following in propositional logic:

No shoes, no shirt, no service
**Definitions:** formula and sequent

A *formula* (or well-formed formula) is a syntactically correct equation in the propositional logic.

\[(p \lor q) \to p\] is a formula

\[p \lor q \neg r\] is not a formula

Assume we have a set of formulas \(\phi_1, \phi_2, \ldots, \phi_n\), which we will call the *premises*. Also assume we have a formula \(\psi\), which we will call the conclusion.

We say that the *sequent*

\[
\phi_1, \phi_2, \ldots, \phi_n \vdash \psi
\]

is *valid* if we can find a proof for \(\psi\) given the premises \(\phi_1, \phi_2, \ldots, \phi_n\). For example,

\[p \lor q, \neg q \vdash p\]

is valid.
Elementary Proof Rules:

- **and-introduction:** \( \land i \)

\[
\begin{array}{c}
\phi \\
\psi \\
\hline
\phi \land \psi \\
\hline
\phi \\
\hline
\phi \land \psi \\
\end{array}
\]

- **and-elimination:** \( \land e \)

\[
\begin{array}{c}
\phi \land \psi \\
\hline
\phi \\
\hline
\phi \land \psi \\
\psi \\
\hline
\phi \\
\hline
\psi \\
\end{array}
\]

or

Use the above proof rules to show that the following sequents are valid:

\( p \land q, r \vdash q \land r \)

\( (p \land q) \land r, s \land t \vdash q \land s \)
• **double negation-elimination**: \( \neg\neg\phi \rightarrow \phi \)

\[
\begin{array}{c}
\neg\phi \\
\hline \\
\neg\neg\phi \\
\hline \\
\phi \\
\end{array}
\]

• **double negation-introduction**: \( \neg\neg\phi \rightarrow \phi \)

\[
\begin{array}{c}
\phi \\
\hline \\
\neg\neg\phi \\
\hline \\
\neg\phi \\
\end{array}
\]

Prove the following sequent is valid

\[ p, \neg\neg (q \land r) \vdash \neg\neg p \land r \]
• implication-elimination: \( \rightarrow e \)
(also known as Modus Ponens, MP)

\[
\begin{align*}
\phi & \quad \phi \rightarrow \psi \\
\hline
& \quad \psi \rightarrow e
\end{align*}
\]

Formal terminology for implication is, for the formula \( \phi \rightarrow \psi \), the proposition \( \phi \) is the \textit{antecedent} and the proposition \( \psi \) is the \textit{consequent}. The rule Modus Ponens is also known as “affirming the antecedent”, because we are affirming that \( \psi \) is true.

Note that \( \rightarrow e \) can be applied to complex sentences, e.g.,

1. \( (p \land q) \rightarrow (q \lor r) \) premise
2. \( p \land q \) premise
3. \( q \lor r \rightarrow e \) 2, 1

• denying the consequent, or \textit{Modus Tollens}: MT

\[
\begin{align*}
\phi & \quad \psi \quad \neg \psi \\
\hline
\phi \rightarrow \psi & \quad \neg \psi \quad \text{MT}
\end{align*}
\]

This rule asserts that, for a given implication, from the falsity of the consequent we can infer the falsity of the antecedent.

As with Modus Ponens, this rule can be used for complex sentences.
Prove that the following sequents are valid:
\[ p \rightarrow (q \rightarrow r), p, \neg r \vdash \neg q \]
\[ \neg p \rightarrow \neg q, q \vdash p \]
\[ p \rightarrow \neg q, q \vdash \neg p \quad \text{(take home problem)} \]

Express the following in propositional logic:

- Today it will rain or shine but not both.
- My sister wants a black and white cat.
- No one will pass the course who does not study.
- John and Mary both went to the show; John liked the show but Mary did not.