Please write your answer to each question only in the box marked Answer.
No questions will be answered by the instructors during the exam.
This is a closed-book exam. No computers or calculators are allowed.
If you are unsure of what is wanted for a particular question, make a reasonable assumption and write this at the beginning of your answer.

NAME: ________________________________

STUDENT NUMBER: _________________

FOR INSTRUCTOR’S USE ONLY

Question 1: _____ / 10

Question 2: _____ / 10

Question 3: _____ / 10

Question 4: _____ / 10

TOTAL: _____ / 40
Question 1: [10 marks]

Are the following set of formulae consistent? Justify your answer.

\[\forall x(\neg P(x) \rightarrow \forall y(Q(x, y)))\]
\[\exists x(P(x))\]
\[\exists x(\neg P(x))\]
\[\neg \exists x(Q(x, x))\]

Answer:
Question 2: [10 marks]

Translate the following argument into the language of predicate logic, using the given predicate, constant and function symbols, and give a formal proof of validity.

All rational numbers are real numbers. Not all rational numbers are integers. Therefore, not all real numbers are integers.

Dictionary:
Q(x) - x is a rational number;
R(x) - x is a real number;
I(x) - x is an integer.

Answer:
Question 3: [10 marks]

Let $\phi$ be the formula:

$$\exists x P(x) \land \forall x (P(x) \rightarrow (\exists y Q(x,y) \land \exists y \neg Q(x,y)))$$

**Answer:**

a) Find a model $\mathcal{M}$ that satisfies the formula

b) Find a model $\mathcal{M}'$ that does not satisfy the formula
Question 4: [10 marks]

Show the semantic entailment $\forall x \neg \phi \models \neg \exists x \phi$. Justify each step.

Answer: