School of Computing  
CISC204, Test #2  
PAPER A  
October 21, 2015

Please answer only in the answer boxes provided. You may use the back of the pages as scrap paper. **This is a closed-book test. No computers or calculators are allowed.**

A reference page is provided at the end of the test. You may use only these rules of inference.

Should a question be unclear or ambiguous, you should make a *reasonable* interpretation and state what you have assumed.

To be eligible for re-marking, this test must be answered entirely in indelible (unerasable) ink. If erasable ink or pencil is used, then the test will be marked exactly once.

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**Do not begin until instructed to do so.**

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<table>
<thead>
<tr>
<th>Question</th>
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<tbody>
<tr>
<td>1.</td>
<td>15</td>
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<td>2.</td>
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<td>3.</td>
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Total: /40  
**Test Mark:** /40
1. This question is a test of understanding the semantics of a formula in propositional logic. For the formula $\phi$ that is

\[(\neg p \land q) \lor (p \land \neg q)) \land \neg r\]

(a) Construct the truth table for $\phi$

(b) Construct the conjunctive Normal Form (CNF) for $\phi$

You may use either the truth table or the conversion algorithm. Show all of your steps.

15 points
2. This question is a test of translating natural language into the language of predicate logic. For this question, use these predicate symbols, function symbols, and constants:

\[ D(x): \text{x is a student} \]
\[ E(x): \text{x is an exam} \]
\[ P(x, y): \text{x is a problem on exam y} \]
\[ S(x, y): \text{x solves y} \]
\[ f(x, y): \text{the } x^{th} \text{ problem on exam y} \]
\[ h(x): \text{the hardest problem on exam x} \]

1: one
2: two
3: three

Translate each of the following sentences into predicate logic.

(a) No student solves problem one on exam three

(b) Every student who solves problem one on exam one also solves the hardest problem on exam one

(c) If some student solves every problem on every exam, then some student solves the hardest problem on every exam

(d) Every student solves some problem on every exam

(e) There is no student who solves every problem on some exam

10 points
3. This question is a test of understanding the scope of variables in predicate logic. Consider the formula
\[ \exists x (P(x) \land Q(y) \rightarrow \forall z R(x, z)) \]

(a) Draw the parse tree for \( \phi \)

(b) Identify every free variable and every bound variable in \( \phi \)
   You may simply state the results; description of your method is optional.

(c) Compute the substitution \( \phi[g(a, z)/y] \)

10 points
4. This question is a test of a proof that uses the predicate rules =i and =e, in addition to the rules of propositional logic. Prove the sequent

\[(y = 0) \land (y = x) \vdash 0 = x\]

Justify each step of your proof.

**Answer:**
The basic rules in propositional logic:

<table>
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<tr>
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</table>
| \( \land \) | \[ \begin{array}{c}
\phi \\
\psi
\end{array} \] \( \land i \) | \[ \begin{array}{c}
\phi \\
\psi
\end{array} \] \( \land e_1 \) | \[ \begin{array}{c}
\phi \\
\psi
\end{array} \] \( \land e_2 \) |
| \( \lor \) | \[ \begin{array}{c}
\phi
\end{array} \] \( \lor i_1 \) | \[ \begin{array}{c}
\psi
\end{array} \] \( \lor i_2 \) | \[ \begin{array}{c}
\phi \\
\psi
\end{array} \] \( \lor e \) |
| \( \rightarrow \) | \[ \begin{array}{c}
\phi \\
\vdots \\
\psi
\end{array} \] \( \rightarrow i \) | \[ \begin{array}{c}
\phi
\end{array} \] \( \rightarrow e \) |
| \( \bot \) | (none) | \[ \begin{array}{c}
\bot
\end{array} \] \( \bot e \) |
| \( \neg \) | \[ \begin{array}{c}
\phi
\vdots \\
\bot
\end{array} \] \( \neg i \) | \[ \begin{array}{c}
\phi
\vdots \\
\bot
\end{array} \] \( \neg e \) |

Some derived rules in propositional logic:

\( \phi \rightarrow \psi \quad \neg \psi \)

\[ \begin{array}{c}
\phi
\end{array} \]

\( \neg \phi \) \( \rightarrow i \)

\( \phi \quad \neg \phi \)

\[ \begin{array}{c}
\neg \phi
\vdots \\
\bot
\end{array} \]

\( \phi \) \( \neg \neg i \)

\( \phi \lor \neg \phi \) \( \lor i \)

\[ \begin{array}{c}
\phi
\end{array} \]

\( \phi \lor \neg \phi \) \( \lor e \)

Substitution rules in predicate logic:

\( t_1 = t_2 \quad \phi[t_1/x] \)

\[ \begin{array}{c}
t = t
\end{array} \]

\( \phi[t_2/x] \)

\[ \begin{array}{c}
t_1 = t_2 \quad \phi[t_1/x] = e
\end{array} \]

\[ \begin{array}{c}
t_2 = t_2 \quad \phi[t_2/x] = e
\end{array} \]