School of Computing
CISC/CMPE 204
Logic In Computer Science

Test # 1, Paper A

October 4, 2016

Please answer only in the answer boxes provided. You may use the back of the pages as scrap paper.

This is a closed-book test. No computers or calculators are allowed.

A reference page is provided at the end of the test. You may use only these rules of inference.

Should a question be unclear or ambiguous, you should make a reasonable interpretation and state what you have assumed.

To be eligible for re-marking, this tests must be answered entirely in indelible (unerasable) ink. If erasable ink or pencil is used, then the test will be marked exactly once.

Do not begin until instructed to do so.

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<thead>
<tr>
<th>Question</th>
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<td>Question 1</td>
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<td>Question 2</td>
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**Question 1** - This question is a test of proving the validity of a simple sequent. Prove the sequent

$$\neg p \rightarrow q, \neg r \rightarrow \neg q \vdash \neg p \rightarrow r$$

Justify each step of your proof.
Question 2 - This question is a test of proving the validity of a more complicated sequent. Prove the sequent

\[ p \lor q, p \rightarrow r, \neg s \rightarrow \neg q \vdash r \lor s \]

Justify each step of your proof.

Answer

10 points
**Question 3** - This question tests your ability to establish logical equivalence. Consider the two formulas

\[ p \land (q \lor r) \quad \text{and} \quad q \lor (p \land r) \]

If these formulas are equivalent, prove the equivalence using the rules of deduction.

If they are not equivalent, provide a model of \( \{p, q, r\} \) such that one formula is true and the other is false.

If they are equivalent, justify each step of your proof; if they are not, provide a complete truth table.

**Answer**

10 points
**Question 4** - This question is a test of a complicated proof. Prove the sequent

\[ s \lor (r \land \neg q), (s \lor r) \rightarrow (p \lor \neg q) \vdash q \rightarrow p \]

Justify each step of a proof.

**Answer**

5 points
**Reference: Rules of Deduction**

The basic rules:

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Some derived rules:

- **Modus Tollens (MT)**: \[\phi \rightarrow \psi, \neg \psi \rightarrow \neg \phi\]
- **Double Negation (\(\neg\neg\))**: \[\neg \neg \phi \rightarrow \phi\]
- **Peano-Brouwer Continuity (PBC)**: \[\bot \rightarrow \phi\]
- **Law of Excluded Middle (LEM)**: \[\phi \lor \neg \phi\]