School of Computing  
CISC/CMPE 204  
Logic In Computer Science  

Test # 2, Paper A  

October 18, 2016

Please answer only in the answer boxes provided. You may use the back of the pages as scrap paper.  
This is a closed-book test. No computers or calculators are allowed.  
A reference page is provided at the end of the test. You may use only these rules of inference.  
Should a question be unclear or ambiguous, you should make a reasonable interpretation and state what you have assumed.  
To be eligible for re-marking, this tests must be answered entirely in indelible (unerasable) ink. If erasable ink or pencil is used, then the test will be marked exactly once.

Do not begin until instructed to do so.

For Marker Use Only

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Question 1 - This question is a test of understanding the semantics of a formula in propositional logic. For the formula $\phi$ that is

$$\left( (\neg q \land p) \lor (q \land \neg p) \right) \land r$$

Justify each step of your proof.

(a) Construct the truth table for $\phi$

(b) Construct the conjunctive Normal Form (CNF) for $\phi$
You may use either the truth table or the conversion algorithm. Show all of your steps.

15 points
Question 2 - This question is a test of translating natural language into the language of predicate logic. For this question, use these predicate symbols, function symbols, and constants:

D(x): x is a student
E(x): x is an exam
P(x, y): x is a problem on exam y
S(x, y): x solves y
f(x, y): the x\textsuperscript{th} problem on exam y
h(x): the hardest problem on exam x
1: one
2: two
3: three

Translate each of the following sentences into predicate logic.

(a) No student solves problem three on exam one

(b) Every student who solves problem one on exam two also solves the hardest problem on exam two

(c) If some student solves every problem on every exam, then some student solves the hardest problem on every exam

(d) No student solves every problem on every exam

(e) If some student solves no problems on exam one, then some student will not solve the hardest problem on exam one

10 points
**Question 3** - This question is a test of understanding the scope of variables in predicate logic. Consider the formula

\[ \exists x (P(x, y) \lor Q(y, z) \rightarrow \forall z R(z, x)) \]

(a) Draw the parse tree for \( \phi \)

(b) Identify every free and bound variable in \( \phi \)

(c) Complete the substitution \( \phi[g(a, z)/y] \)

10 points
Question 4 – This question deals with the concept of Horn Satisfiability. Is the following Horn Clause satisfiable?

\[ \phi = (T \rightarrow x) \land (T \rightarrow s) \land (z \rightarrow \bot) \land (w \land x \land q \rightarrow z) \land (w \rightarrow q) \land (T \rightarrow v) \land (v \rightarrow w) \]

Show your work step by step and justify your conclusion.

**Answer**

5 points
Reference: Rules of Deduction

The basic rules in propositional logic:

<table>
<thead>
<tr>
<th></th>
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| ∧ | \[
\frac{\phi \land \psi}{\phi \land \psi
\]
\] | \[
\frac{\phi \land \psi}{\phi
\]
\] | \[
\frac{\phi \land \psi}{\phi
\]
\] | \[
\frac{\phi \land \psi}{\phi
\]
\] |
| ∨ | \[
\frac{\phi \lor \psi}{\phi \lor \psi}
\]
\] | \[
\frac{\phi \lor \psi}{\phi \lor \psi}
\]
\] | \[
\frac{\phi \lor \psi}{\phi \lor \psi}
\]
\] | \[
\frac{\phi \lor \psi}{\phi \lor \psi}
\]
\] |
| → | \[
\frac{\phi \rightarrow \psi}{\phi \rightarrow \psi}
\]
\] | \[
\frac{\phi \rightarrow \psi}{\phi \rightarrow \psi}
\]
\] | \[
\frac{\phi \rightarrow \psi}{\phi \rightarrow \psi}
\]
\] | \[
\frac{\phi \rightarrow \psi}{\phi \rightarrow \psi}
\]
\] |
| ⊥ | (none) | \[
\frac{\phi}{\bot}
\]
\] | \[
\frac{\phi}{\bot}
\]
\] | \[
\frac{\phi}{\bot}
\]
\] | \[
\frac{\phi}{\bot}
\]
\] |
| ¬ | \[
\frac{\phi}{\neg \phi}
\]
\] | \[
\frac{\phi}{\neg \phi}
\]
\] | \[
\frac{\phi}{\neg \phi}
\]
\] | \[
\frac{\phi}{\neg \phi}
\]
\] |

Some derived rules in propositional logic:

\[
\frac{\phi \rightarrow \psi \quad \neg \psi}{\neg \phi
\]
\] (MT)

\[
\frac{\neg \phi}{\neg \phi
\]
\] (¬ i)

\[
\frac{\neg \phi}{\phi
\]
\] (PBC)

\[
\frac{\phi \lor \neg \phi}{\phi
\]
\] (LEM)

Substitution rules in predicate logic:

\[
\frac{t_1 = t_2}{\phi[t_1/x] = \phi[t_2/x]
\]
\] (e)