CISC 204 Class 17

Test #1: Normal Forms, Basic Predicate Logic

There were two versions of this test, “A” and “B”, because the classroom did not have enough space for there to be an empty seat between students.

The questions for version “A” of the test were:

1. For the formula \( \phi \) that is
   \[ (\neg p \land q) \lor (p \land \neg q) \lor \neg r \]
   (a) Construct the truth table for \( \phi \)
   (b) Construct the conjunctive Normal Form (CNF) for \( \phi \)

2. Use these predicate symbols, function symbols, and constants: 
   \( D(x) \): \( x \) is a student; 
   \( E(x) \): \( x \) is an exam; 
   \( P(x, y) \): \( x \) is a problem on exam \( y \); 
   \( S(x, y) \): \( x \) solves \( y \); 
   \( f(x, y) \): the \( x \)th problem on exam \( y \); 
   \( h(x) \): the hardest problem on exam \( x \); 
   \( 1 \): one; 
   \( 2 \): two; 
   \( 3 \): three.

   Translate each of the following sentences into predicate logic:
   (a) No student solves problem one on exam three
   (b) Every student who solves problem one on exam one also solves the hardest problem on exam one
   (c) If some student solves every problem on every exam, then some student solves the hardest problem on every exam
   (d) Every student solves some problem on every exam
   (e) There is no student who solves every problem on some exam

3. Consider the formula
   \[ \exists x (P(x) \land Q(y) \rightarrow \forall z R(x, z)) \]
   (a) Draw the parse tree for \( \phi \)
   (b) Identify every free variable and every bound variable in \( \phi \)

4. Prove the sequent
   \[ (y = 0) \land (y = x) \vdash 0 = x \]