CISC 204  Class 28

Review of Material

Test #4 in this course will cover predicate semantics. The material will be from Week 8 and Week 9 of the course. The main concepts are:

- Semantics: extending propositional logic with a universe of discourse
- Models: Predicates as sets, functions
- Logical environments: assignments of free variables
- Semantic entailment: satisfiability, validity, entailment

To study for this test, students should be able to do at minimum these exercises from the textbook:

§ 2.4: 1 3 5 6 7 10 11(d) 12(a) 12(b) 12(e)

A learning outcome is that students should be able to find counterexamples to formulas. In this context, students should be able to find models in which these sentences are not true:

\[ \phi_1 \overset{\text{df}}{=} \exists x \, \exists y \, \neg (x = y) \]
\[ \phi_2 \overset{\text{df}}{=} \forall x \, \forall y \, (x = y) \]
\[ \phi_3 \overset{\text{df}}{=} \exists x \, \forall y \, \neg (x = y) \]

Students should also be able to answer: is sentence \( \phi_3 \) ever true?

The two pages of reference material from Test #3 will be reproduced on Test #4. Students are expected to understand the rules and how to apply them.

The next page is a sample answer for the textbook Exercise 7 in § 2.4; this question was on previous tests and may be a useful study aid.
Example: show that $\exists x \neg \phi \models \neg \forall x \phi$

Sample argument:

Let $\mathcal{M}$ be any model, with an assignment to the variable $t$, in which the formula $\neg \phi[t/x]$ holds. The conclusion is that it is false that, for any environment that assigns a value to $x$ in $\mathcal{M}$, the formula $\phi$ holds. This can be shown by contradiction.

Assume, in the same model $\mathcal{M}$, that for any value assigned to $x$, the formula $\phi$ holds. Then the formula $\phi[t/x]$ holds, by the principle of variable substitution.

This contradicts the premise, so the assumption is false. Therefore, it is false that, for all values of $x$ in $\mathcal{M}$, the formula $\phi$ holds.