Now that we have the basics of semantics for predicate logic, and a way of talking about the natural numbers, we can re-visit the problem of translating a symbolic formula to English, and English to symbolic logic.

The problem of translation is so difficult that there are no known algorithms that solve it. Consequently, we are left with human ingenuity when faced with this sometimes vexing problem. Here are some heuristics for dealing with connectives and quantifiers.

Beginning with connectives in propositional logic, when people write “and” they usually mean conjunction. Other conjunctives, in both parsing and in translation, are “however”, “although”, “even though”, and “despite”.

The word “but” almost always is conjunction that emphasizes a semantics interpretation that the reader might find surprising. The word “but” can be tricky because it also has poetic and colloquial connotations; a famous sentence attributed to Benjamin Franklin begins

“For the want of a nail the shoe was lost . . .”

and is sometimes rendered as

“But for a nail the shoe was lost . . .”

In these sentences, the idea is that a solidly implanted horse-shoe has solidly implanted nails, and the absence (negation) of a nail had the consequence of the absence of a horse-shoe. This is Modus Tollens in prose, so the word “but” carries an implication with it.

People are less consistent when they write “or”, so great care must be taken. It usually means inclusive (ordinary) disjunction and sometimes means exclusive disjunction. For example, when asking about coffee we will usually enquire “cream or sugar” which is inclusive. When we ask “coffee or tea”, we are definitely meaning an exclusive disjunction. In a song’s lyrics, “should I stay or should I go”, exclusive disjunction is more clearly meant.

The word “unless” requires some thought in translation. It usually means exclusive disjunction, such as “I’ll be in my office unless I get a call from the hospital”; the meaning is that either the speaker will be in the office or will be attending to consequences of a phone call. Sometimes “unless” is a hidden implication, such as “I’ll be at the meeting unless the Provost is there”.

The words “neither” and “nor” are, by default, inclusive disjunctions that negate the nouns or noun phrases that they connect. The phrase “neither rain nor snow” is equivalent to “not rain and not snow”, by De Morgan’s law.
The term “not both” is denying a conjunction, logically equivalent to “neither/nor”; “not both $p$ and $q$” is symbolically $\neg (p \land q)$. The term “both not” asserts the conjunction of negations, so “both not $p$ and $q$” is symbolically $\neg p \land \neg q$.

There are many ways of expressing logical implication in English. Some words that are used are “if”, “only if”, “hence”, “therefore”, “thus”, “follows from”, “just when”, and “then”.

More complex are the words “so” and “as”, because the semantics of the sentence matter a great deal in a translation. An important reading of these words is to be synonymous with “therefore” and “thus”, where what follows is a conclusion; a prudent translation is as a sequent using the symbol $\vdash$. For example, we might render “I like tea and coffee, so I like tea” as

$$T \land C \vdash T$$

Managing quantification is even more complicated than managing connectives.

The words “all” and “any” are usually hints that universal quantification is involved in a translation. The word “no” can also indicate universal quantification, with caution; the sentence “no cats are dogs” is an example that must be treated carefully.

In the scope of a universal quantifier, it is usually a good idea to interpret conjugations of the verb “to be” as implication. We began this topic by considering the sentence “All humans are mortal”; using these heuristics, it is best to translate this as

$$\forall x (H(x) \rightarrow M(x))$$

where the predicate $H$ is for “human” and $M$ is for “mortal”. It would be a mistake to translate this as a conjunction; the symbolic sentence

$$\forall x (H(x) \land M(x))$$

would assert “everything in the universe of discourse is both human and is mortal”, which is not the intended meaning.

The sentence “no cats are dogs” is an assertion of universal exclusive disjunction, which can be rendered in many equivalent ways. Sticking to the custom of preferring implication in this context, we can read the sentence as an abbreviation for “no cat is a dog and no dog is a cat”, which can be translated as

$$\forall x ((C(x) \rightarrow \neg D(x)) \land (D(x) \rightarrow \neg C(x)))$$

The words “some”, “someone”, “something”, “somewhere” and related constructs usually mean existential quantification. The word “no” can also indicate existential quantification, so care must be taken. The sentence “nobody goes to that restaurant anymore” does not mean, literally, that for all persons that person does not go; it means that the restaurant is less
frequented by a subset of the universe of discourse, such as important people who formerly
went there (with the sub-text that, because the restaurant is still in business, some people still
 go there – just not the people in the subset of interest).

In the scope of an existential quantifier, it is usually a good idea to interpret conjugations of
the verb “to be” as conjunction. In our introductory example, “Socrates is human” has the
intended meaning that there is something in the universe of discourse that is both Socrates
and is human. The preferred translation is

$$\exists x \ (S(x) \land H(x))$$

To see why this is preferred, consider the formula $S(x) \rightarrow H(x)$. From propositional logic,
this is equivalent to $\neg S(x) \lor H(x)$; quantified, and translated into English, this would be

“There is something that is either not Socrates or is human”

which does not have at all the same meaning of the original English sentence.

A generally good practice is that, if a logically equivalent formula seems to be nonsense or
is very different from the first translation, then the first translation is suspect.

These heuristics are for general English. Poetry, or other complex text, is much harder to
translate into symbolic logic. A classic example is a slight updating of a line from Shake-
speare’s “The Merchant of Venice”, which in modern language is

“All that glitters is not gold”

This sentence, poetically, is saying in clearer terms “not all that glitters is gold”, or “it is
false that everything that glitters is gold”, or “there is something that glitters and that is not
gold”.

A beautiful inversion of this sentence is in Tolkein’s “The Lord of the Rings”, which is

“All that is gold does not glitter”

by which the author means “there is something that is gold and that does not glitter”. This is
an allusion to Shakespeare that an erudite reader would at once recognize.