1. Verify the validity of the following correctness statements by adding all the intermediate assertions (that is, give the proof tableau). All variables are of type int. Clearly state any mathematical facts and inference rules used.

   (a) (2 marks)
   
   \[
   \text{ASSERT( } i \leq j \text{ )}
   \]
   \[
   x = i;
   \]
   \[
   y = j;
   \]
   \[
   \text{ASSERT( } y \geq x \&\& y \geq j \&\& ( y = x \mid\mid y = j ) \text{ )}
   \]

   (b) (3 marks)
   
   \[
   \text{ASSERT( } y \geq 0 \&\& z \geq 0 \text{ )}
   \]
   \[
   \text{if ( } y > z \text{ ) } \{ y = y - z; \}
   \]
   \[
   \text{else } \{ z = z - y; \}
   \]
   \[
   \text{ASSERT( } y \geq 0 \&\& z \geq 0 \text{ )}
   \]

2. (5 marks) Write and verify a program that computes the sum of the cubes of the first \( n \) positive integers. Below is the specification.

   Declarative interface:

   ```
   \text{const int n; \text{/* the program will compute the sum of the cubes of}}
   \text{the first n positive integers */}}
   \text{int sum; \text{/* the sum of the cubes is stored in this variable */}}
   ```

   Below are given the pre- and post-condition:
   \[
   \text{ASSERT( } n \geq 1 \text{ )}
   \]
   \[
   \text{/* the program goes here */}
   \]
   \[
   \text{ASSERT( sum == } \sum_{i=1}^{n} i*i*i \text{ )}
   \]

   The program should use a while-loop and standard arithmetic operations.

   Select a loop invariant and give a complete proof tableau for the program (including all the intermediate assertions). Also make an argument for termination.
Regulations on assignments

- The assignments may be done in groups consisting of one, two, three or four students. If more than one student are collaborating on an assignment, they must submit a single joint solution.
- Clearly print (or type) the name(s) and student number(s), and course number, at the top of the first page. Additionally each student collaborating on an assignment must sign the top of the first page.
- If the submission consists of more than one page, the pages must be stapled together.
- Note: You are asked to write your solutions using non-erasable pen (or to type the solutions). Solutions written in pencil or erasable ink will be marked, but they will not be considered for remarking after the assignments are returned.