1. The solution for Question 1 is given in Figure 1 (next page).

2. Question 2 solution: The state diagram has exactly one accepting state which is not the starting state. Hence no initial modification of the state diagram is needed.

   The result of eliminating the 3rd state (that is not starting state or accepting state) is given in Figure 2 (next page). The resulting regular expression obtained from this figure is

   \[(ac^*c)^*(c + ac^*b)(b + ac^*b + ac^*c)(ac^*c)^*(c + ac^*b))^*\]

3. (a) The language \( A \) is regular. It is denoted by the regular expression:

   \[ab + a^2b^2 + a^3b^3 + a^3a^*b^4b^*\]

   (b) We prove that \( B \) is not regular:

   Assume to the contrary that \( B \) is regular and let \( n \) be the constant given by the pumping lemma (\( n \) is the “pumping length”). Consider the string

   \[x = c^n d^n \in B.\]

   \( x \) has length greater than \( n \) and hence \( x \) can be split into three parts \( x = p \cdot q \cdot r \) where \( p, q, r \) satisfy the conditions given in the pumping lemma.

   According to the pumping lemma, \( |pq| \leq n \) and \( q \neq \varepsilon \). Since \( pq \) is a prefix of \( x \), it follows that \( pq \) is a prefix of \( c^n \) and this means that \( q = c^i \) for some \( i \geq 1 \). According to the pumping lemma, the string \( pq^0r = pr \) should be in \( B \). However, \( pr = c^{n-i}d^n \).

   (Note that \( pr \) is obtained from \( x = pqr \) by “removing” a substring consisting of \( i \) symbols \( c \).) This means that \( pr \not\in B \) which is a contradiction.

   Thus the assumption that \( B \) is regular leads to a contradiction, and we have proved that \( B \) is not regular.
Figure 1: Solution for Question 1. In the figure, “e” denotes the empty string “ε”.

Figure 2: Intermediate stage in the solution of Question 2.