INSTRUCTIONS

- **Aids allowed:** You may bring in one 8.5 × 11 inch sheet of paper containing notes, and use it during the exam. The sheet can be written/printed on both sides.

- This examination is THREE HOURS in length. Answer all 10 questions.

- **Answer each question in the space provided (on the question paper).** There are two extra pages at the end of the exam, if more space is needed. Please write legibly.

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

STUDENT NUMBER: ________________

STUDENT NUMBER (written in words):

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1. Consider the context-free grammar

\[ S \rightarrow aaSbbb \mid aaS \mid Sbbb \mid \varepsilon \]

Here, S is the start nonterminal, and a, b are terminals.

(i) Give a parse tree for the string \( aaaaabbb \) (= \( a^4 b^3 \)).

(ii) Is the above grammar ambiguous? Justify your answer!

(iii) Eliminate left-recursion from the grammar using the general method (grammar transformation) for eliminating left-recursion.

(iv) Give a parse tree for the string \( aabbbbbb \) (= \( a^2 b^6 \)) using your new grammar (that has no left-recursion).

(v) Give a regular expression that denotes the language generated by this grammar.
2. In this question the alphabet is $\Sigma = \{a, b\}$. For each of the following sets of strings over $\Sigma$, give a regular expression that denotes it.

   (i) All strings over $\{a, b\}$ that have substring $abba$.
      Regular expression:

   (ii) All strings over $\{a, b\}$ that have an odd number of occurrences of the symbol $b$.
      Regular expression:

   (iii) All strings of even length over $\{a, b\}$ that have at least one $b$.
      Regular expression:

   (iv) All strings over $\{a, b\}$ that do not have substring $aaa$.
      Regular expression:

3. Give context-free grammars that generate the following languages $L_1$ and $L_2$.

   (i) $L_1 = \{a^i b^k c^m \mid m = i + k, \ i \geq 0, k \geq 0, m \geq 0\}$

   (ii) $L_2 = \{a^{2i+1} b^{3i+2} c^{4k+3} d^{3k+2} \mid i \geq 0, k \geq 0\}$
4. Verify the validity of the following correctness statements by adding all the intermediate assertions (that is, give the proof tableau). All variables are of type int. Clearly state any mathematical facts and inference rules used.

(i) \text{ASSERT( } x < 1 \text{ \&\& } y == 5 \text{ )}
\text{x = x - y;}
\text{y = y - x;}
\text{x = x + y;}
\text{ASSERT( x == 5 \&\& y > 7 )}

(ii) \text{ASSERT( } z < 0 \text{ )}
\text{if ( x < y ) \{ x = y - z; \} // end-if}
\text{y = x - y;}
\text{ASSERT( y > z )}
5. (i) (2 marks) What should the pre-condition P be in each of the following correctness statements for the statement to be an instance of Hoare’s axiom scheme? All variables are of type int.

(a) P { \( x = y + 2z; \) } ForAll(\( z = 0; z < x \)) \( 2x + w \geq 3z \)

(b) P { \( z = x + z; \) } Exists(\( z = 0; z < y \)) \( x+z \geq y*z \)

(ii) (3 marks) Use the array-component assignment axiom (two times) to find the most general sufficient pre-condition P for the following code fragment:

\[ \text{ASSERT(P) /*determine what is P*/} \]
\[ \text{A[i] = A[k];} \]
\[ \text{A[k] = x;} \]
\[ \text{ASSERT( A[i] == A[k] + 1 )} \]

Above A is an array of integers, x is an integer variable and we assume that all the subscripts are within the range of subscripts for A.

Write the assertion P first using the notation from the array-component assignment axiom, and then rewrite P in a logically equivalent form that does not contain any notation \( \langle A \mid I \mapsto E \rangle \).
6. Are the following languages $A$ and $B$ over the alphabet $\Sigma = \{a, b, c, d\}$ regular or non-regular?

- If a language is regular, give a regular expression that denotes it.
- If a language is non-regular, prove that it is not regular.

(i) $A = \{a^i b^k c^\ell d^m \mid i + k = \ell + m, \ i \geq 0, \ k \geq 0, \ \ell \geq 0, \ m \geq 0 \}$

(ii) $B = \{b^{2i+3} c^{3k-2} d^{4m+3} \mid i \geq 1, \ k \geq 1, \ m \geq 1 \}$
7. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) Let $L$ be the language consisting of all strings over the alphabet $\{b, c\}$ having an equal number of $b$'s and $c$'s. The language $L$ is denoted by the regular expression:

(a) $(b + c)^* + (bb + cc)^*$
(b) $(bc + cb + bbcc + bebc + bccb + cbcb + cccb + cccb)^*$
(c) $((bc + cb)^* + (bbcc + bebc + bccb + cbcb + cccb + cccbb))^*$
(d) None of the above.

(ii) What is the language generated by the grammar $S \to bSccc \mid bS \mid \varepsilon$ where $\alpha$ derives the empty string. Which of the following conditions always prevents the use of recursive-descent parsing with this grammar:

(a) $\text{first}(S) \cap \text{first}(\beta) \neq \emptyset$
(b) $\text{follow}(S) \cap \text{first}(\beta) \neq \emptyset$
(c) $\text{follow}(S) \cup \text{first}(\alpha) \neq \emptyset$
(d) None of the above conditions necessarily prevents the use of recursive-descent parsing.

(iii) What should $x$ be in \[
\frac{Q_1 \land \& \land B(C) \land Q_2}{Q_1 \land \{ & b \} \land Q_2}
\] in order to make this a valid inference rule for if-statements “if (B) C”?

(a) $x$ should be: $(Q_1 \land \& \land B) \mid Q_2$
(b) $x$ should be: $(Q_1 \land \& \land B) \implies Q_2$
(c) $x$ should be: $Q_2 \implies (Q_1 \land \& \land B)$
(d) None of the above choices gives a valid inference rule.

(iv) Consider an inference rule for correctness statements: $\frac{Q_1(C)P_1}{Q_1 \land \& \land C \land Q_2(C)} \frac{Q_2(C)P_2}{Q_2 \land \& \land C \land P_2}$. This inference rule is:

(a) Generally valid.
(b) Valid only if $C$ has no assignments to variables used in the assertions $P_1$ and $P_2$.
(c) Valid only if $C$ does not interfere with variables used in the assertions $Q_1$ and $Q_2$.
(d) None of the above.

(vi) The Church-Turing thesis states that

(a) The halting problem cannot be solved using programming languages (such as C) but it can be solved using Turing machines.
(b) Turing machines cannot solve the halting problem.
(c) Functional programming languages can implement certain functions that cannot be computed by Turing machines.
(d) None of the above.
8. (i) (2 marks) Using left-factoring and/or elimination of left-recursion give grammars equivalent to the below two grammars where the immediate problems preventing use of recursive-descent parsing have been removed. Capital letters denote variables and the set of terminals is \{a, b, c, d\}.

(a) \( S \to dcSd \mid daS \mid db \mid abSb \mid ba \)

(b) \( S \to bc \mid Sbc \mid dcS \mid Scd \mid Sadd \)

(ii) (3 marks) Consider the context-free grammar with the following productions (here capital letters are nonterminals and \( S \) is the start nonterminal):

\[
S \to AbB \mid d \\
A \to aAb \mid cB \mid \varepsilon \\
B \to cAd \mid dA \mid \varepsilon
\]

(a) (1 mark) Determine the sets:

- follow(\( S \)) = \ldots
- follow(\( A \)) = \ldots
- follow(\( B \)) = \ldots

(b) (2 marks) Does the grammar allow the use of recursive-descent parsing? Justify your answer.
9. In this question the alphabet is $\Sigma = \{a, b, c\}$.

   (i) Using the systematic method described in the course, convert the below state diagram into an equivalent state diagram without $\varepsilon$-transitions.

   ![State Diagram](image)

   (ii) Using the systematic method described in the course (subset construction), convert the below nondeterministic state diagram into a deterministic state diagram. Your answer should indicate how the deterministic state diagram is obtained from the nondeterministic one: the states of the deterministic diagram should be labeled by sets of states of the nondeterministic diagram.

   ![State Diagram](image)
10. Assume a declarative interface where \( n \) and \( \text{max} \) are constant integers. Also \( A \) is an array of integers and we know that the entries in the segment \( A[0: \text{max}] \) are defined.

Consider the following (partial) correctness statement:

```c
ASSERT(1 <= n < \text{max})
{
    int j; j = n-1;
    A[n] = n;
    while (j > 0) { A[j] = A[j+1] + j;
        j = j-1; }
    //end-while
}
ASSERT( \text{ForAll}(k = 1; k < n+1) \ A[k] == (n-k+1)*(n+k)/2 )
```

Choose a **loop invariant** and give a complete proof tableau by adding all the **intermediate assertions**. Be sure to clearly **indicate what is your loop invariant**. Also state any mathematical facts used. Does the loop terminate? Explain your answer.
Do all 10 questions. Student#:  

1st extra page.
2nd extra page.