Queen’s University  
CISC-223 1st Midterm February 2014

INSTRUCTIONS

• You have forty-five minutes. Attempt all five questions.

• You may bring in one 8.5 × 11 sheet of paper containing notes, and use it during the midterm.

• Answer each question in the space provided (on the question paper). There is an extra page at the end of the exam if more space is needed. Please write legibly.

• Please note: You are asked to write your answers using a non-erasable pen. Answers written in pencil or erasable ink will be marked, but they will not be considered for remarking after the midterms are returned.

NAME: ____________________________ (optional)

STUDENT NUMBER: ________________ (required)

MARKS

| Problem 1 | /6 |
| Problem 2 | /5 |
| Problem 3 | /6 |
| Problem 4 | /7 |
| Problem 5 | /6 |
| Total     | /30 |
1. Consider the following three languages over the alphabet $\Sigma = \{a, b\}$:
   
   $E = \{b, aab\}$, $F = \{bab, \varepsilon\}$, $G = \{abba\}$. List explicitly the elements, if any, of the following languages:

   (i) $E + F + G$

   (ii) $E \cdot F$

   (iii) $E \cdot (F + G)$

   (iv) $E^2$

   (v) $E + \{\varepsilon\} + \emptyset$

   (vi) $(E + F + G) \cdot (\emptyset)^*$

2. (i) (2 marks) Give a **regular expression** that defines the set of all strings over $\Sigma = \{c, d\}$ that have an even number of $c$’s. (Note: zero is an even number.)

   (ii) (3 marks) Draw a **deterministic state transition diagram** that recognizes the set of all strings over $\Sigma = \{a, b\}$ that have an odd number of $a$’s and an odd number of $b$’s.
3. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) Which of the following statements is true:
   (a) Some deterministic state diagrams do not have an equivalent regular expression.
   (b) Some nondeterministic state diagrams do not have an equivalent regular expression.
   (c) Some regular expressions do not have an equivalent deterministic state diagram.
   (d) The above three statements are all false.

(ii) Let \( L \) be the language defined by the regular expression \((a + aab + aaab)^* \cdot (aa + aaa)\).
Which of the following strings is in \( L \):
   (a) \( aaaabaa \)
   (b) \( aaabboa \)
   (c) \( aabaaab \)
   (d) None of the above strings is in \( L \).

(iii) Which of the following regular expressions defines the set of all strings over \( \{a, b\} \) that have at least 2 times as many \( a \)'s as \( b \)'s:
   (a) \( a^*(aab + aba + baa)^*a^* \)
   (b) \( (a + aab + aba + baa)^* \)
   (c) \( a^*(a*aaba*abaa* + ba*aa)^*a^* \)
   (d) None of the above.

(iv) Which of the following statements is true:
   (a) Every regular language is finite.
   (b) Every regular language is infinite.
   (c) Some regular languages are finite and some are infinite.
   (d) None of the above.

(v) Let \( A \) and \( B \) be regular languages over an alphabet \( \Sigma \). The following is true:
   (a) \( A \cap B \) is always regular.
   (b) \( A \cap B \) is never regular.
   (c) \( A \cap B \) is sometimes, but not always, regular.
   (d) None of the above.

(vi) Which of the following statements is true for all languages \( B \). Here \( \subseteq \) denotes “is a subset or equal to”:
   (a) \( B \subseteq B \cdot B \)
   (b) \( B \cdot B \subseteq B \)
   (c) \( B \cdot \emptyset \subseteq B \)
   (d) \( B \subseteq B \cdot \emptyset \)
   (e) None of the above.
4. (i) Using the algorithm discussed in class minimize the number of states of the below DFA. The alphabet is $\Sigma = \{a, b\}$. Your answer should clearly indicate how you arrived at the solution. Draw the minimized state diagram where each state is labeled by the names of states that were merged together.

(ii) Using the systematic method described in the course (subset construction), convert the below nondeterministic state diagram into a **deterministic** state diagram. Your answer should indicate how the deterministic state diagram is obtained from the nondeterministic one: the states of the deterministic diagram should be labeled by sets of states of the nondeterministic diagram.
5. Are the below languages $L_1$, $L_2$ over alphabet $\Sigma = \{a, b, c, d\}$ regular or nonregular?

- If a language is regular, give a regular expression that defines it.
- If a language is not regular, prove using the pumping lemma that it is not regular.

(i)  $L_1 = \{a^{3i}b^{2k}c^m \mid i \geq 0, \; k \geq 0, \; m \geq 0\} \cdot \{a^r b^s \mid r \geq 0, \; s \geq 0\}$

Here “$\cdot$” is the concatenation of languages.

(ii) $L_2 = \{b^j c^k d^m \mid j \geq 0 \text{ and } k \geq m \geq 0\}$