1. (2 marks) Let $\Sigma = \{0, 1\}$ and consider the languages $A$ and $B$ over $\Sigma$ where $A = \{1, 10, 010\}$, $B = \{1, 01\}$

(a) Write down all strings in the set $A \cdot B$. How many strings there are in $A \cdot B$?
(b) Write down all strings in the set $B \cdot A$. How many strings there are in $B \cdot A$?

2. (3 marks) In this question the alphabet is $\Sigma = \{a, b\}$. Let $R = (ab + aab)^*a^*$ and $S = (a^*ba^*)^*a^*$.

(a) Give an example of a string $z$ that is both in $R$ and in $S$ (that is, $z \in R \cap S$).
(b) Give an example of a string $x$ that is in $R$ and is not in $S$ (that is, $x \in R \cap \overline{S}$ where $\overline{S}$ is the complement of $S$).
(c) Give an example of a string $y$ that is in $S$ and is not in $R$ (that is, $y \in \overline{R} \cap S$).

3. (5 marks) Show how to define the following five languages over $\Sigma = \{0, 1\}$ using only $\varepsilon$, the alphabet symbols 0 and 1, and the operations of union, concatenation, and closure.

Note: Your answer cannot use the intersection or complementation operation.
Below “or” always means “inclusive or”.

(a) All strings that have 11011 as a substring.
(b) All strings that have prefix 000 or have suffix 111.
(c) All strings of even length. Note that zero is an even number.
(d) All strings that have both 01 and 10 as substrings. Note that the substrings can occur in either order and possibly overlap.
(e) All strings that begin with 0 and end with 0 and do not have 010 as a substring.
Regulations on assignments

- **The assignments may be done in groups consisting of one, two, three or four students.** If more than one student are collaborating on an assignment, they must submit a single **joint solution**.

- At the top of the first page, for each student collaborating on the assignment, type or write in **clear capital letters** the following information:
  - **LAST-NAME, FIRST-NAME** (name as it appears on solus, e.g., “SMITH, NANCY”)
  - the student number (e.g., “1234 4321”)
  - “CISC 223” or “CMPE 223” (depending on which course you are in)
  - signature (the signature need not be easily readable)

  The information for each one student should be written on one line and in the order specified above.

- **Bonus mark:** Papers that have the above information, for all the participants, written exactly correctly and perfectly clearly and legibly will receive one bonus mark. The assignment is worth 10 marks. Papers that receive the bonus mark, may get more than 10 marks. For the bonus mark there is no partial credit for incomplete information or unclear handwriting.

- The assignment should be put into the locked CISC 223 drop-off box on the 2nd floor of Goodwin hall by the due date. The assignments must be submitted in hardcopy. Assignments sent by email are not accepted.

- If the submission consists of more than one page, the pages must be **stapled** together.

- **Note:** You are asked to write your solutions using non-erasable pen (or to type the solutions). Solutions written in pencil or erasable ink will be marked, but they will not be considered for remarking after the assignments are returned.