One bonus mark for neatly written student information: Papers that have all the names and student numbers written exactly as requested in the regulations (found at the end), will receive one bonus mark.

1. (4 marks) Use the array-component assignment axiom two times to find the (most general) sufficient pre-condition \( P \) for the following code fragment:

\[
\text{ASSERT( } P \text{ )} /* \text{determine what is } P */
\]

\[
\begin{align*}
A[j] &= x; \\
A[i] &= A[k]; \\
\text{ASSERT( } A[j] == 2 \text{ )}
\end{align*}
\]

Above \( x \) is an integer variable, \( A \) is an array of integers and we assume that all the subscripts are within the range of subscripts for \( A \).

First write the assertion \( P \) using the notation from the array-component assignment axiom, and then rewrite \( P \) in a logically equivalent and simplified form that does not contain any notation \((A | I \mapsto E)\) (as in examples on pp. 85–86 and in examples covered in class).

2. (6 marks) Assume a declarative interface where \( n \) and \( \text{max} \) are constant integers. Also \( A \) is an array of integers and we know that the entries in the segment \( A[0:\text{max}] \) are defined. For nonnegative integers \( x \) and \( i \), \( \text{power}(x,i) \) denotes \( x^i \) (the \( i \)’th power of \( x \)), that is, \( \text{power}(x,0) = 1 \) and \( \text{power}(x,i+1) = x \ast \text{power}(x,i) \). (\( * \) denotes multiplication.)

Consider the following (partial) correctness statement:

\[
\text{ASSERT(1 <= n < max)}
\]

\[
\begin{align*}
\{ \text{int } j; & \ j = n-2; \\
& A[n-1] = 1; \\
& \text{while ( } j >= 0 \text{ ) } \{ \ A[j] = 2\ast A[j+1] + 1; \\
& \quad j = j - 1; \} \ //\text{end-while} \\
\}
\]

\[
\text{ASSERT( ForAll( } k = 0; \ k < n \text{ ) } A[k] == \text{power}(2, n-k) - 1 \ )
\]

Choose a loop invariant and give a complete proof tableau by adding all the intermediate assertions. Be sure to clearly indicate what is your loop invariant. Also state any mathematical facts used. Does the loop terminate? Explain your answer.
Regulations on assignments

- **The assignments may be done in groups consisting of one, two, three or four students.** If more than one student are collaborating on an assignment, they must submit a single **joint solution.**

- At the top of the first page, for each student collaborating on the assignment, type or write in **clear capital letters** the following information:
  - LAST-NAME, FIRST-NAME (name as it appears on solus, e.g., “SMITH, NANCY”)
  - the student number (e.g., “1234 4321”)
  - “CISC 223” or “CMPE 223” (depending on which course you are in)
  - signature (the signature need not be easily readable)

  The information for each one student should be written on one line and in the order specified above.

- **Bonus mark:** Papers that have the above information, for all the participants, written exactly correctly and perfectly clearly and legibly will receive a bonus mark. The assignment is worth 10 marks. Papers that receive the bonus mark, may get more than 10 marks. For the bonus mark there is no partial credit for incomplete information or unclear handwriting.

- The assignment should be put into the locked CISC 223 drop-off box on the 2nd floor of Goodwin hall by the due date. The assignments must be submitted in hardcopy. Assignments sent by email are not accepted.

- If the submission consists of more than one page, the pages must be **stapled** together.

- **Note:** You are asked to write your solutions using non-erasable pen (or to type the solutions). Solutions written in pencil or erasable ink will be marked, but they will not be considered for remarking after the assignments are returned.