Queen’s University
CISC/CMPE-223 1st Practice Midterm February 2018

INSTRUCTIONS

• You have 40 minutes. Attempt all four questions.

• You may bring in one 8.5 × 11 sheet of paper containing notes, and use it during the midterm.

• Answer each question in the space provided (on the question paper). There is an extra page at the end of the exam if more space is needed. Please write legibly.

• Please note: You are asked to write your answers using a non-erasable pen. Answers written in pencil or erasable ink will be marked, but they will not be considered for remarking after the midterms are returned.

STUDENT NUMBER: ________________________________

Student number (written in words):

______________________________

CIRCLE YOUR COURSE:  CISC-223  CMPE-223 (IMPORTANT)

MARKS

<table>
<thead>
<tr>
<th>Problem</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>/5</td>
</tr>
<tr>
<td>Problem 2</td>
<td>/5</td>
</tr>
<tr>
<td>Problem 3</td>
<td>/6</td>
</tr>
<tr>
<td>Problem 4</td>
<td>/5</td>
</tr>
<tr>
<td>Total</td>
<td>/21</td>
</tr>
</tbody>
</table>
1. Consider the following three languages over the alphabet $\Sigma = \{a, b\}$:

$$A = \{a, ba\}, \quad B = \{\varepsilon, bbb\}, \quad C = \{\varepsilon\}.$$

List explicitly the elements, if any, of the following languages:

(i) $A + B$

(ii) $A + C$

(iii) $A \cdot B$

(iv) $A \cdot B \cdot C$

(v) $A^2$

(vi) $C^3$

(vii) $C^*$

(viii) $\emptyset^*$
2. In this question \( \Sigma = \{a, b, c, d\} \). Using the systematic method described in the course convert the below state diagram into an equivalent regular expression. Your answer should include the intermediate step(s) used in the construction.
3. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) Which of the following statements is true:
   (a) Every regular language is finite.
   (b) Every regular language is infinite.
   (c) Some regular languages are finite and some are infinite.
   (d) None of the above.

(ii) Let \( L \) be the language denoted by the regular expression \( a^*b(a + ba^*b)^* \)
Which of the following strings is in \( L \):
   (a) \( bbaab \in L \)
   (b) \( bbaabb \in L \)
   (c) \( aabb \in L \)
   (d) None of the above strings is in \( L \).

(iii) Which of the following regular expressions defines the set of all strings over \( \{b, c\} \) having an odd number of \( c \)'s:
   (a) \( (b + c)(bb + bc + cb + cc)^* \)
   (b) \( b^*c(b + cb)c^* \)
   (c) \( b^*c(b^* + c^*b^*c^*)^*(b^* + \varepsilon) \)
   (d) None of the above.

(iv) Let \( K = \{a^ib^k \mid i \geq k \geq 0\} \). The language \( K \) is denoted by the regular expression:
   (a) \( (aa + a + b)^* \)
   (b) \( aaa^*(bbb^* + bb^* + b^*) + aa^*(bb^* + b^*) \)
   (c) \( (aaa)^*(bbb)^*(bb)^* + (aa)^*((bb)^* + b^*) + a^*b^* \)
   (d) None of the above.

(v) Let \( A \) and \( B \) be regular languages over an alphabet \( \Sigma \). The following is true:
   (a) \( A \cap B \) is always regular.
   (b) \( A \cap B \) is never regular.
   (c) \( A \cap B \) is sometimes, but not always, regular.
   (d) None of the above.

(vi) Let \( A \) and \( B \) be languages where \( A \subseteq B \) (\( A \) is a subset of \( B \)). The following implication holds:
   (a) If \( A \) is regular, then \( B \) is regular.
   (b) If \( B \) is regular, then \( A \) is regular.
   (c) If \( A \) is non-regular, then \( B \) is non-regular.
   (d) None of the above.
4. Are the below languages $A$ and $B$ over alphabet $\Sigma = \{a, b, c, d\}$ regular or nonregular?

- If a language is regular, give a regular expression that defines it.
- If a language is not regular, prove using the pumping lemma that it is not regular.

(i) $A = \{ a^{2i}b^{3i} \mid i \geq 1 \} \cup \{ c^{2j+1}d^{3k+2} \mid j \geq 1, \; k \geq 1 \}$
(Here “$\cup$” is the union of languages.)

(ii) $B = \{ a^{i+1}b^k \mid i \geq 0, \; k \geq 0 \} \cup \{ b^{2r+1}c^{3s} \mid r \geq 0, \; s \geq 0 \}$
(Here “$\cup$” is the union of languages.)