1. Consider the following three languages over the alphabet $\Sigma = \{a, b\}$:

$$A = \{a, ba\}, \quad B = \{\varepsilon, bbb\}, \quad C = \{\varepsilon\}.$$

List explicitly the elements, if any, of the following languages:

(i) $A + B$ \hspace{1cm} \{ $a, ba, \varepsilon, bbb$ \}

(ii) $A + C$ \hspace{1cm} \{ $a, ba, \varepsilon$ \}

(iii) $A \cdot B$ \hspace{1cm} \{ $a, ba, a b b b, b a b b b$ \}

(iv) $A \cdot B \cdot C$ \hspace{1cm} \{ $a, ba, a b b b, b a b b b$ \}

(v) $A^2$ \hspace{1cm} \{ $a a, a b a, b a a, b a b a$ \}

(vi) $C^3$ \hspace{1cm} \{ $\varepsilon$ \}

(vii) $C^*$ \hspace{1cm} \{ $\varepsilon$ \}

(viii) $\emptyset^*$ \hspace{1cm} \{ $\varepsilon$ \}
2. In this question $\Sigma = \{a, b, c, d\}$. Using the systematic method described in the course convert the below state diagram into an equivalent regular expression. Your answer should include the intermediate step(s) used in the construction.

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Regex:
$$(ac^*b)^*(ac^*a)(dc^*a + (b + dc^*b)) \
\cdot (ac^*b)^*(ac^*a)^*$$
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3. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) Which of the following statements is true:
   (a) Every regular language is finite.
   (b) Every regular language is infinite.
   (c) Some regular languages are finite and some are infinite.
   (d) None of the above.

(ii) Let $L$ be the language denoted by the regular expression $a^*b(a + ba^*b)^*$
Which of the following strings is in $L$:
   (a) $bbaab \in L$
   (b) $bbaabb \in L$
   (c) $aabb \in L$
   (d) None of the above strings is in $L$.

(iii) Which of the following regular expressions defines the set of all strings over \{b, c\} having an odd number of c's:
   (a) $(b + c)(bb + bc + cb + cc)^*$
   (b) $b^*c(b + cb^*)^*$
   (c) $b^*c(b^* + c^*b^*c^*)^*(b^* + \epsilon)$
   (d) None of the above.

(iv) Let $K = \{a^ib^k \mid i \geq k \geq 0\}$. The language $K$ is denoted by the regular expression:
   (a) $(aa + a + b)^*$
   (b) $aaa^*($\(bb^* + bb^* + b^*\) + $aa^*(bb^* + b^*)$
   (c) $(aaa)^*(($\(bb^*)^* + ($bb^* + b^* + a^*b^*$
   (d) None of the above.

(v) Let $A$ and $B$ be regular languages over an alphabet $\Sigma$. The following is true:
   (a) $A \cap B$ is always regular.
   (b) $A \cap B$ is never regular.
   (c) $A \cap B$ is sometimes, but not always, regular.
   (d) None of the above.

(vi) Let $A$ and $B$ be languages where $A \subseteq B$ ($A$ is a subset of $B$). The following implication holds:
   (a) If $A$ is regular, then $B$ is regular.
   (b) If $B$ is regular, then $A$ is regular.
   (c) If $A$ is non-regular, then $B$ is non-regular.
   (d) None of the above.
4. Are the below languages A and B over alphabet $\Sigma = \{a, b, c, d\}$ regular or nonregular?

- If a language is regular, give a regular expression that defines it.
- If a language is not regular, prove using the pumping lemma that it is not regular.

(i) $A = \{a^{2i}b^{3i} \mid i \geq 1\} \cup \{c^{2j+1}d^{3k+2} \mid j \geq 1, k \geq 1\}$
   (Here "\cup" is the union of languages.)

(ii) $B = \{a^{i+1}b^{k} \mid i \geq 0, k \geq 0\} \cup \{b^{2r+1}c^{3s} \mid r \geq 0, s \geq 0\}$
   (Here "\cup" is the union of languages.)

\textbf{i) A is nonregular:}

Assume that A is regular and let $n$ be the constant given by the P.L. Choose $x = a^{2n}b^{3n} \in A$.

By the P.L. we can write $x = pqr$ where the parts $p, q, r$ satisfy the conditions of the P.L.

Now $pq$ is a prefix of $a^{2n}$ and $q \neq \varepsilon$. Thus, $q = a^z, z \geq 1$, and we get $pqr = a^{2n+z}b^{3n} \notin A$ because $\frac{2n+z}{3n} \neq \frac{2}{3}$.

This is a contradiction and shows that A is nonregular.

\textbf{ii) Regex for B:}

$$a^*b^* + b(b^2)^*(c^3)^*$$