1. (i) (2 marks) What should the pre-condition P be in each of the following correctness statements for the statement to be an instance of Hoare’s axiom scheme? All variables are of type int.

(a) \[ P \{ x = 2y + z; \} \quad y \geq xw + 37 \]

\[ y \geq (2y + z) * w + 37 \]

(b) \[ P \{ x = 2y + z; \} \quad \text{Exists}(x = 0; \ x < 500) \ x * y \leq v + 55 \]

\[ \exists x (x = 0; \ x < 500) \ x * y \leq v + 55 \]

(ii) (3 marks) Verify the validity of the following correctness statement by adding all the intermediate assertions (that is, give the proof tableau). All variables are of type int. Clearly state any mathematical facts and inference rules used.

\[
\text{ASSERT}( \ x > 4 \ & \ x = 7 )
\]

\[
\text{x = x - y;}
\]

\[
\text{y = y - x;}
\]

\[
\text{x = x + y;}
\]

\[
\text{ASSERT}( \ x = 7 \ & \ x < 10 )
\]

Solution:

\[
\text{ASSERT}( \ x > 4 \ & \ y = 7 ) \quad \text{// y = 7 \ & \ x > 4 \text{ implies } 2y - x < 10}
\]

\[
\text{ASSERT}( \ y = 7 \ & \ 2y - x < 10 ) \quad \text{// arithmetic simplification}
\]

\[
\text{x = x - y;}
\]

\[
\text{ASSERT}( \ x+y-x = 7 \ & \ y - x < 10 )
\]

\[
\text{y = y - x;}
\]

\[
\text{ASSERT}( \ x+y = 7 \ & \ y < 10 )
\]

\[
\text{x = x + y;}
\]

\[
\text{ASSERT}( \ x = 7 \ & \ y < 10 )
\]

Note: Main variants of questions included. The questions have further variants and your paper may not be exactly as shown.
1. (i) (2 marks) What should the pre-condition P be in each of the following correctness statements for the statement to be an instance of Hoare's axiom scheme? All variables are of type int.

(a) P \{ x = 2*y + z; \} u + v >= x*w + 19

\[ u + v \geq (2 \times y + z) \times w + 19 \]

(b) P \{ x = y + 2*z; \} Exists(z = 0; z < 500) x*z <= v + 61

\[ \exists z (z = 0; z < 500) (y + 2 \times z) \times a \leq v + 61 \]

(ii) (3 marks) Verify the validity of the following correctness statement by adding all the intermediate assertions (that is, give the proof tableau). All variables are of type int. Clearly state any mathematical facts and inference rules used.

```plaintext
ASSERT( x < 3 && y > 2 )
x = x - y;
y = x + y;
x = x + 2;
ASSERT( x < 2 && y < 3 )
```

Solution:

```plaintext
ASSERT( x < 3 && y > 2 )
// x < 3 && y > 2 implies x-y < 0 when x, y int
ASSERT( x - y < 0 && x - y + y < 3 )
x = x - y;
ASSERT( x < 0 && x + y < 3 )
y = x + y;
ASSERT( x + 2 < 2 && y < 3 )
x = x + 2;
ASSERT( x < 2 && y < 3 )
```
1. (i) (2 marks) What should the pre-condition \( P \) be in each of the following correctness statements for the statement to be an instance of Hoare's axiom scheme? All variables are of type \( \text{int} \).

(a) \( P \{ y = x + 3 * z; \} \) \( z * y \geq x * w + 37 \)

\[
2 * (x + 3 * z) \geq x * w + 37
\]

(b) \( P \{ y = 2 * x + z; \} \) \( \text{Exists}(x = 0; \ x < 500) \) \( x * y \leq v + 55 \)

\[
\text{Exists}(x = 0; \ x < 500) \quad 2 * (2 * x + z) \leq v + 55
\]

(ii) (3 marks) Verify the validity of the following correctness statement by adding all the intermediate assertions (that is, give the proof tableau). All variables are of type \( \text{int} \). Clearly state any mathematical facts and inference rules used.

\begin{verbatim}
ASSERT( x >= 2 \ |\ x <= -2 )
y = 2*x;
z = x - y;
y = x + z;
ASSERT( x+y >= 1 \ |\ z >= 2 )
\end{verbatim}

Solution:

\begin{verbatim}
ASSERT( x >= 2 \ |\ x <= -2 ) // x >= 2 implies x >= 1
    // x <= -2 iff -x >= 2
ASSERT( x >= 1 \ |\ -x >= 2 ) // arithmetic simplification
ASSERT( 3*x - 2*x >= 1 \ |\ x - 2*x >= 2 )
y = 2*x;
ASSERT( 3*x - y >= 1 \ |\ x - y >= 2 )
z = x - y;
ASSERT( 2*x + z >= 1 \ |\ z >= 2 )
y = x + z;
ASSERT( x+y >= 1 \ |\ z >= 2 )
\end{verbatim}
1. (i) (2 marks) Using left-factoring and/or elimination of left-recursion give grammars equivalent to the below two grammars where the immediate problems preventing use of recursive-descent parsing have been removed. Capital letters denote variables and the set of terminals is \( \{a, b, c, d\} \).

(a) \( S \rightarrow Sdc \mid Sab \mid ccd \mid \varepsilon \)

\[
S \rightarrow cc \; S' \mid S' \\
S' \rightarrow d \; cs' \mid ab \; S' \mid \varepsilon
\]

(b) \( S \rightarrow abSa \mid bbcSab \mid bbaSc \mid cac \mid dd \)

\[
S \rightarrow bbs' \mid aabSa \mid cac \mid dd \\
S' \rightarrow cSa \mid ab \; S
\]

(ii) (3 marks) Consider the language \( L = \{a^{2k}b^{2i}c^i \mid i \geq 1, \; k \geq 1\} \).

Draw a **deterministic** pushdown automaton that recognizes the language \( L \).
1. (i) (2 marks) Using left-factoring and/or elimination of left-recursion give grammars equivalent to the below two grammars where the immediate problems preventing use of recursive-descent parsing have been removed. Capital letters denote variables and the set of terminals is \{a, b, c, d\}.

(a) \[ S \rightarrow S \alpha d \mid Scba \mid Sdc \mid \epsilon \]

\[ S \rightarrow S' \]
\[ S' \rightarrow \alpha d S' \mid cba S' \mid dc S' \mid \epsilon \]

(b) \[ S \rightarrow cba S \mid cbb S a \mid bbb S c \mid acc \mid \epsilon \]

\[ S \rightarrow cbs S' \mid bbs S c \mid acc \mid \epsilon \]
\[ S' \rightarrow \alpha S \mid bSa \]

(ii) (3 marks) Consider the language \( L = \{b^{2i}c^i d^{2k} \mid i \geq 1, k \geq 1\}\)

Draw a deterministic pushdown automaton that recognizes the language \( L \).
4. Are the below languages $A$ and $B$ over alphabet $\Sigma = \{a, b, c, d, f\}$ context-free or non-context-free?

- If a language is context-free, give a **context-free grammar** that defines it.
- If a language is not context-free, **prove using the pumping lemma** that it is not context-free.

(i) $A = \{a^{2i+1}b^k c^{2i+1}d^{2i+1} \mid i \geq 0, k \geq 0\}$

(ii) $B = \{b^i c^{2i} d^{2k} \mid i \geq 1, k \geq 1\} \cup \{c^m d^{3m} f^m a^t \mid m \geq 1, t \geq 1\}$

(Here "\cup" is the union of languages.)

(i) **$A$ is not context-free.** Assume $A$ is CF and let $p$ be the pumping length (PL). Choose $s = a^{2p+1}c^{2p+1}d^{2p+1}$. By the PL, we can write $s = uvwxy$ where the parts satisfy the conditions of the PL.

(i) If $v$ or $x$ have occurrences of more than one symbol, $uv^2wx^2y \notin \Sigma^*$ and is not in $A$.

(ii) Grammar for $B$: $S \rightarrow X \mid Y$

- $X \rightarrow UV$
- $U \rightarrow bUc^2 \mid bc^2$
- $V \rightarrow d^2V \mid d^2$
- $Y \rightarrow WZ$
- $W \rightarrow cWd^3 \mid cd^3$
- $Z \rightarrow fZA \mid fa$
4. Are the below languages \( A \) and \( B \) over alphabet \( \Sigma = \{a, b, c, d, f\} \) context-free or non-context-free?

- If a language is context-free, give a context-free grammar that defines it.
- If a language is not context-free, prove using the pumping lemma that it is not context-free.

(i) \( A = \{ a^{2i}b^{3i+1}c^{2k+1}d^{k+2} \mid i \geq 1, \ k \geq 1 \} \)

(ii) \( B = \{ b^i c^j d^{2i} \mid i \geq 0 \} \cup \{ c^m d^{3m} f^t \mid m \geq 1, \ t \geq 1 \} \)
   (Here "\( \cup \)" is the union of languages.)

1) Grammar for \( A \):
   \[
   S \rightarrow XY \\
   X \rightarrow a^2Xb^3 \mid a^2b^4 \\
   Y \rightarrow c^2Yd \mid c^3d^3
   \]

2) We show that \( B \) is not CF. Assume \( B \) is CF and let \( p \) be the constant of the pumping lemma. Choose \( s = b^p c^p d^{2p} \).
   By the PL we can write \( s = uvwxy \) where the parts satisfy assumptions of the PL.

2a) If \( v \) or \( x \) contains occurrences of more than one symbol, \( uv^2wx^2y \) will have symbols in wrong order and is not in \( B \).

2b) If \( v \) and \( x \) each have occurrences of at least one symbol, \( uv^2wx^2y = b^{p+i}c^{p+j}d^{2p+k} \) where exactly one of \( i, j, k \) are non-zero. This means that number of occurrences of \( b, c, d \) do not have ratios \( 1:1:2 \) and \( uv^2wx^2y \notin B \).
2. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) What is the language generated by the grammar \( S \rightarrow aSbb \mid aSb \mid \varepsilon \)

(a) \( \{a^i b^2k \mid 0 \leq i \leq k\} \)
(b) \( \{a^i b^k \mid 0 \leq i \leq k \leq 2i\} \)
(c) \( \{a^i b^k \mid 0 \leq k \leq 2i\} \)
(d) None of the above.

(ii) Which of the following statements is true:

(a) There exists a language \( L \) recognized by a nondeterministic pushdown automaton such that \( L \) cannot be recognized by any deterministic pushdown automaton.
(b) There exists a language \( L \) recognized by a deterministic pushdown automaton such that \( L \) cannot be recognized by any nondeterministic pushdown automaton.
(c) There exists a regular language \( L \) such that \( L \) cannot be recognized by any deterministic pushdown automaton.
(d) The above three statements are all false.

(iii) The inference rule \( \frac{\text{P(C)} \land Q}{\text{P(C)} \implies R} \) is called

(a) Pre-condition strengthening
(b) Pre-condition weakening
(c) Post-condition weakening
(d) None of the above

(iv) Consider the correctness statement: \( x \geq 0 \{ x = x - 1; \} Q \) where \( x \) has type integer. The correctness statement is valid when \( Q \) is the assertion:

(a) \( x > 0 \)
(b) \( x < 0 \)
(c) \( x \neq 0 \)
(d) None of the above.

(v) Consider the following correctness statement: \( \text{false} \{ \text{C} \} \text{false} \)

(a) The correctness statement is valid for any terminating code \( \text{C} \).
(b) The correctness statement is invalid for any terminating code \( \text{C} \).
(c) The correctness statement is valid for some terminating code \( \text{C} \) and invalid for other terminating code \( \text{C} \).
(d) None of the above.
3. For each question, **circle one answer**. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) What is the language generated by the grammar \( S \rightarrow bScc | bS | \varepsilon \)

(a) \( \{b^i c^{3k} \mid 0 \leq i \leq k \} \)

(b) \( \{b^i c^{3k} \mid 0 \leq k \leq i \} \)

(c) \( \{b^i c^k \mid 0 \leq 3k \leq i \} \)

(d) None of the above.

(ii) Consider the language \( L_1 = \{\alpha b^k \mid i \geq 0, \ k \geq 0 \} \).

(a) The language \( L_1 \) is regular and context-free.

(b) The language \( L_1 \) is context-free but not regular.

(c) The language \( L_1 \) is regular but not context-free.

(d) None of the above.

(iii) Consider a context-free grammar that has productions \( S \rightarrow \alpha | \beta \). Which of the following conditions prevents the use of recursive-descent parsing with this grammar:

(a) \( \text{first}(\alpha) \cup \text{first}(\beta) \neq \emptyset \)

(b) \( \text{first}(\alpha) \cap \text{first}(\beta) = \emptyset \)

(c) \( \text{first}(\alpha) \cap \text{first}(\beta) \neq \emptyset \)

(d) None of the above conditions necessarily prevents the use of recursive-descent parsing.

(iv) The inference rule \( \frac{R(C)Q}{P \implies R} \) is called

(a) Pre-condition strengthening

(b) Pre-condition weakening

(c) Post-condition strengthening

(d) None of the above

(v) Consider the correctness statement: \( x = x \{ \ x = x+1; \ \} \ P \)

where \( x \) has type integer. The correctness statement is valid when \( P \) is the assertion:

(a) \( x > 0 \)

(b) \( x < 0 \)

(c) \( \text{false} \)

(d) None of the above.
3. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) What is the language generated by the grammar \( S \to dS \)cc | \( ddS \) | \( \varepsilon \)
   
   (a) \( \{d^i c^k \mid 0 \leq 2k \leq 3i\} \)
   (b) \( \{d^i c^k \mid 0 \leq k \leq i\} \)
   (c) \( \{d^i c^k \mid i \geq 0, \ k \geq 0\} \)
   (d) None of the above.

(ii) Which of the following classes consists of exactly all the context-free languages:
   
   (a) The languages accepted by deterministic pushdown automata.
   (b) The languages accepted by nondeterministic pushdown automata.
   (c) The languages that can be parsed using recursive-descent parsing.
   (d) None of the above.

(iii) Consider a context-free grammar that has productions \( S \to \alpha \mid \beta \) where \( \alpha \) derives the empty string. Which of the following conditions always prevents the use of recursive-descent parsing with this grammar:
   
   (a) \( \text{first}(S) \cap \text{first}(\beta) \neq \emptyset \)
   (b) \( \text{follow}(S) \cap \text{first}(\beta) \neq \emptyset \)
   (c) \( \text{follow}(S) \cup \text{first}(\alpha) \neq \emptyset \)
   (d) None of the conditions prevents the use of recursive-descent parsing.

(iv) What should \( X \) be in \( \frac{\text{If} \ Q \ \text{then} \ (P \ \text{and} \ B)}{\text{if} (B) \ C} \) in order to make this a valid inference rule for if-statements “if (B) C”
   
   (a) \( X \) should be: \( Q \) implies \( (P \ \text{and} \ B) \)
   (b) \( X \) should be: \( Q \) implies \( (P \ \text{and} \ \neg B) \)
   (c) \( X \) should be: \( (P \ \text{and} \ \neg B) \) implies \( Q \)
   (d) None of the above choices gives a valid inference rule.

(v) Consider the following correctness statement: \( \text{true} \ \{C\} \ \text{false} \)
   
   (a) The correctness statement is valid for any terminating code \( C \).
   (b) The correctness statement is invalid for any terminating code \( C \).
   (c) The correctness statement is valid for some terminating code \( C \) and invalid for other terminating code \( C \).
   (d) None of the above.
3. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) What is the language generated by the context-free grammar

\[ S \to aSbb \mid Sb \mid \epsilon \]

(a) \( \{a^i b^k \mid 0 \leq 3i \leq k\} \)
(b) \( \{a^i b^k \mid 0 \leq k \leq 3i\} \)
(c) \( \{a^i b^{3k} \mid 0 \leq i \leq k\} \)
(d) None of the above.

(ii) Which of the following properties does not always prevent using a recursive-descent parser constructed directly from the given grammar productions:

(a) The grammar is ambiguous.
(b) The grammar has two productions \( A \to w_1, A \to w_2 \), where strings \( w_1 \neq w_2 \) begin with the same terminal symbol.
(c) The grammar has left-recursive productions.
(d) All of the above properties always prevent the use of recursive-descent parsing.

(iii) Let \( P \) and \( Q \) be any assertions. The following holds always:

(a) \( (P \text{ is stronger than } Q) \text{ or (} Q \text{ is stronger than } P \) \)
(b) \( (P \ || \ Q) \) is stronger than \( P \)
(c) \( (P \ &\& \ Q) \) is stronger than \( Q \)
(d) None of the above statements holds generally.

(iv) Consider the correctness statement: \( P \{ x = x*x + 1; \} x <= 0 \)
where \( x \) has type integer. The correctness statement is valid when \( P \) is the assertion:

(a) \( x > 0 \)
(b) \( x < 0 \)
(c) \( \text{true} \)
(d) \( \text{false} \)
(e) None of the above.

(v) Consider the following correctness statement: \( \text{true} \{ C \} \text{ true} \)

(a) The correctness statement is valid for any terminating code \( C \).
(b) The correctness statement is invalid for any terminating code \( C \).
(c) The correctness statement is valid for some terminating code \( C \) and invalid for other terminating code \( C \).
(d) None of the above.
2. For each question, circle one answer. If you circle more than one answer, it will be considered a wrong answer. If in doubt, it is to your advantage to make a guess.

(i) What is the language generated by the grammar \( S \rightarrow cSdd | cS | \varepsilon \)

(a) \( \{c^i d^k | 0 \leq i \leq k\} \)
(b) \( \{c^i d^k | 0 \leq k \leq i\} \)
(c) \( \{c^i d^k | 0 \leq 3k \leq i\} \)
(d) None of the above.

(ii) Which of the following statements is true:

(a) There exist regular languages that are not context-free.
(b) There exist regular languages that cannot be accepted by deterministic pushdown automata.
(c) There exist context-free languages that cannot be accepted by deterministic pushdown automata.
(d) None of the above.

(iii) Let \( N \) be a nonterminal of a grammar. The set \( \text{follow}(N) \) consists of

(a) all terminal symbols that may begin a string derived from \( N \).
(b) all terminal symbols that may end a string derived from \( N \).
(c) all nonterminal symbols that may end a string derived from \( N \).
(d) None of the above.

(iv) Determine \( \text{follow}(S) \) for the grammar \( (S, A \text{ are nonterminals, } S \text{ is the start nonterminal}) \):

\( S \rightarrow aSAb | \varepsilon, \quad A \rightarrow aAc | \varepsilon \)

(a) \( \text{follow}(S) = \{a, b\} \)
(b) \( \text{follow}(S) = \{a, b, c\} \)
(c) \( \text{follow}(S) = \{a, b, \text{EOS}\} \)
(d) \( \text{follow}(S) = \{a, b, c, \text{EOS}\} \)
(e) None of the above.

(v) Consider the following correctness statement: \text{false \{ C \} true}

(a) The correctness statement is valid for any terminating code \( C \).
(b) The correctness statement is invalid for any terminating code \( C \).
(c) The correctness statement is valid for some terminating code \( C \) and invalid for other terminating code \( C \).
(d) None of the above.