

CISC-365*
Test #3
October 31, 2008

Student Number (Required) _____

Name (Optional) _____

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, but may not be reconsidered after the test papers have been returned.

The test will be marked out of 50.

Question 1	/20
Question 2	/30
TOTAL	/50

Question 1 (20 marks)

You have won the contract to install Wi-fi nodes along a very straight and sparsely populated stretch of road which runs due east and west. There are N houses along the road – each house is identified by its distance from the east end of the road. Your assignment is to install nodes so that each house is no more than 1 kilometer from a node. You want to install **as few nodes as possible**.

a) **(10 marks)** Give a Greedy Algorithm to find an optimal (minimal) set of locations for the wi-fi nodes.

b) **(10 marks)** Prove that the first choice your algorithm makes for a node location is correct (i.e. that there is an optimal solution that contains this location as its first location).

Question 2 (30 marks)

You are happily attending the annual Bagpipe and Ukelele Music Fest. The organizers have published the schedule of performances all through the day, but it is a bit odd: performances can overlap, and may all have different start times and end times. There is a tradition at the BUMF – if you attend a performance, you must be there for the entire performance. You can't arrive late or leave early from any performance.

Your goal is attend as many performances as you can.

For example, the schedule might be

Performance	1	2	3	4	5	6
Start time	9:15	9:18	8:30	10:02	12:12	12:15
End time	10:32	9:46	12:12	12:14	12:17	15:05

In this example, one solution would be to attend Performance 3 and Performance 5. (Note that Performance 5 starts at the same time as Performance 3 ends. This is ok.) A better solution would be to attend Performances 2, 4 and 6.

(Question 2 continues on the two following pages.)

a) **(18 marks)** Here is a Greedy Algorithm:

1. Sort the performances according to END time, so that the performance that ends earliest is the first in the list.
2. Select the first performance in the list
3. Repeat
4. Select the next performance in the list that does not overlap with any performance selected so far.
5. Until there are no more performances to be considered

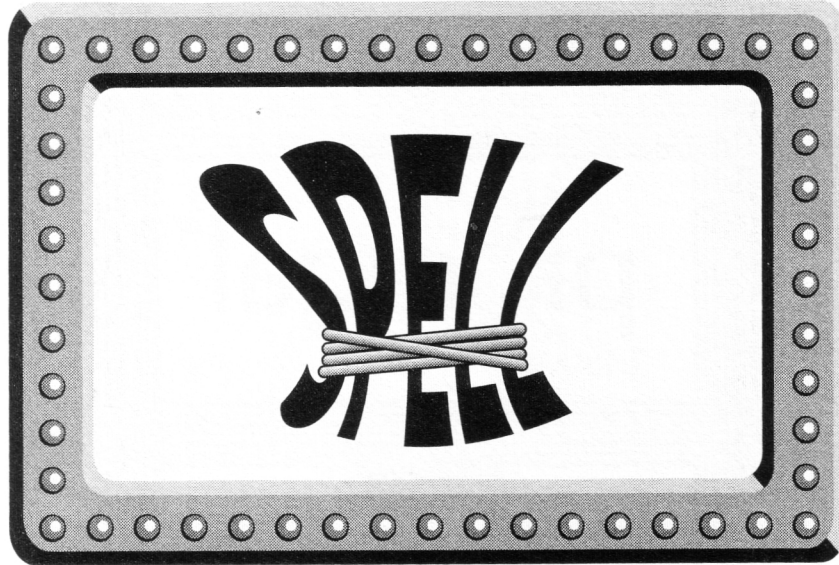
Does this algorithm find an optimal solution for all instances of this problem? Either find a counterexample (a set of performance times for which this algorithm chooses a non-optimal solution) or give a proof that the algorithm is correct. If you are providing a counterexample, show the solution the algorithm would choose and explain why, and show that there is a better solution. If you are proving correctness, you may focus on proving that the algorithm's first choice (Line 2) is appropriate.

b) **(12 Marks)** Here is a different Greedy Algorithm:

1. Sort the performances according to LENGTH, so that the shortest performance is the first in the list
2. Select the first performance in the list
3. Repeat
4. Select the next performance in the list that does not overlap with any performance selected so far
5. Until there are no more performances to be considered

Does this algorithm find an optimal solution for all instances of this problem? Either find a counterexample (a set of performance times for which this algorithm chooses a non-optimal solution) or give a proof that the algorithm is correct. If you are providing a counterexample, show the solution the algorithm would choose and explain why, and show that there is a better solution. If you are proving correctness, you may focus on proving that the algorithm's first choice (Line 2) is appropriate.

Special Bonus Question: (0 marks)



What is the meaning of the figure above?