CISC-462 Asn 1 SOLUTIONS

1. Sequence of config's:

$q_0 abba$
$b q_0 bba$
$q_0 bbba$
$q_0 bbba$
... etc.

(Computation does not halt.)

M operates as follows:

- it rewrites a's to b's moving right
- if M encounters a b, it starts moving toward the left.
- M accepts if it reaches a blank symbol (at right end of input)

M recognizes a^*

2.

a) $q_1 0$

$\sqcup q_2 \sqcup$

$\sqcup \sqcup q_{acc}$

b) $q_1 00$

$\sqcup q_2 0$

$\sqcup x q_3 \sqcup$

$\sqcup q_5 x \sqcup$

$q_5 \sqcup x \sqcup$

$\sqcup q_2 x \sqcup$

$\sqcup x q_2 \sqcup$

$\sqcup x \sqcup q_{acc}$

c) $q_1 000$

$\sqcup q_2 00$

$\sqcup x q_3 0$

$\sqcup x 0 q_4 \sqcup$

$\sqcup x 0 \sqcup q_{rej}$

d) $q_1 000000$

$\sqcup q_2 00000$

$\sqcup x q_3 0000$

$\sqcup x 0 q_4 000$

$\sqcup x 0 x q_3 00$

$\sqcup x 0 x 0 q_4 0$

$\sqcup x 0 x 0 x q_3 \sqcup$

$\sqcup x 0 x 0 q_5 x \sqcup$

$\sqcup x 0 x q_5 0 x \sqcup$

$\sqcup x 0 q_5 x 0 x \sqcup$

$\sqcup x q_5 0 x 0 x \sqcup$

$\sqcup q_5 x 0 x 0 x \sqcup$

$q_5 \sqcup x 0 x 0 x \sqcup$

$\sqcup q_2 x 0 x 0 x \sqcup$

$\sqcup x q_2 0 x 0 x \sqcup$

$\sqcup x x q_3 x 0 x \sqcup$

$\sqcup x x x q_3 0 x \sqcup$

$\sqcup x x x 0 q_4 x \sqcup$

$\sqcup x x x 0 x q_4 \sqcup$

$\sqcup x x x 0 x \sqcup q_{reject}$

3. a) M has the same states as A, and additionally q_{accept}, q_{reject}.

Each transition of the DFA $p \xrightarrow{b} q$ is denoted in the TM-diagram as: $p \xrightarrow{b \to b, R} q$

The start state of the TM is the same as the start state of the DFA.

If q is an accept state of the DFA, then the TM has a transition:

$$q \xrightarrow{\sqcup \to \sqcup, R} q_{accept}$$ (accepts at end of input)

If q is a non-accepting state of the DFA, then the TM has a transition

$$q \xrightarrow{\sqcup \to \sqcup, R} q_{reject}$$ (rejects ~~at~~ end of input)

b)

$$\begin{tikzcd} q_0 \arrow[r, "a \to a{,} R", "b \to b{,} R"'] & \bigcirc \arrow[r, "a \to a{,} R", "b \to b{,} R"'] & \bigcirc \arrow[r, "a \to a{,} R", "b \to b{,} R"'] & \bigcirc \arrow[loop right, "a \to a{,} R \;\; b \to b{,} R"] \arrow[dl, "\sqcup \to \sqcup{,} R"] \\ & & q_{accept} & \end{tikzcd}$$

Undefined transitions go to the reject state.

4. The simulation of a TM M by a two-stack PDA is done as follows:

The non-blank contents of the tape of M is stored on the two stacks. Stack 1 stores the characters on the left of the head, with the bottom of stack storing the leftmost character of the tape of M.

Stack 2 stores the characters to the right of the head, with the bottom of the stack storing the rightmost nonblank character on the tape of TM M.

That is, if M's configuration is $u\,q\,v$,

the two-stack PDA is in state q, with stack 1 storing the string u and stack 2 storing the string v^R (reversal of v), (both from bottom to top).

For each transition $\delta(q, c) = (p, b, L)$ in the TM,

the PDA A pops c off stack 2, pushes b into stack 2, then pops stack 1 and pushes the character into stack 2, and goes from state q to p.

For each transition $\delta(q, c) = (p, b, R)$ in the TM,

the corresponding PDA transition pops c off stack 2, and pushes b into stack 1, and goes from state q to state p.

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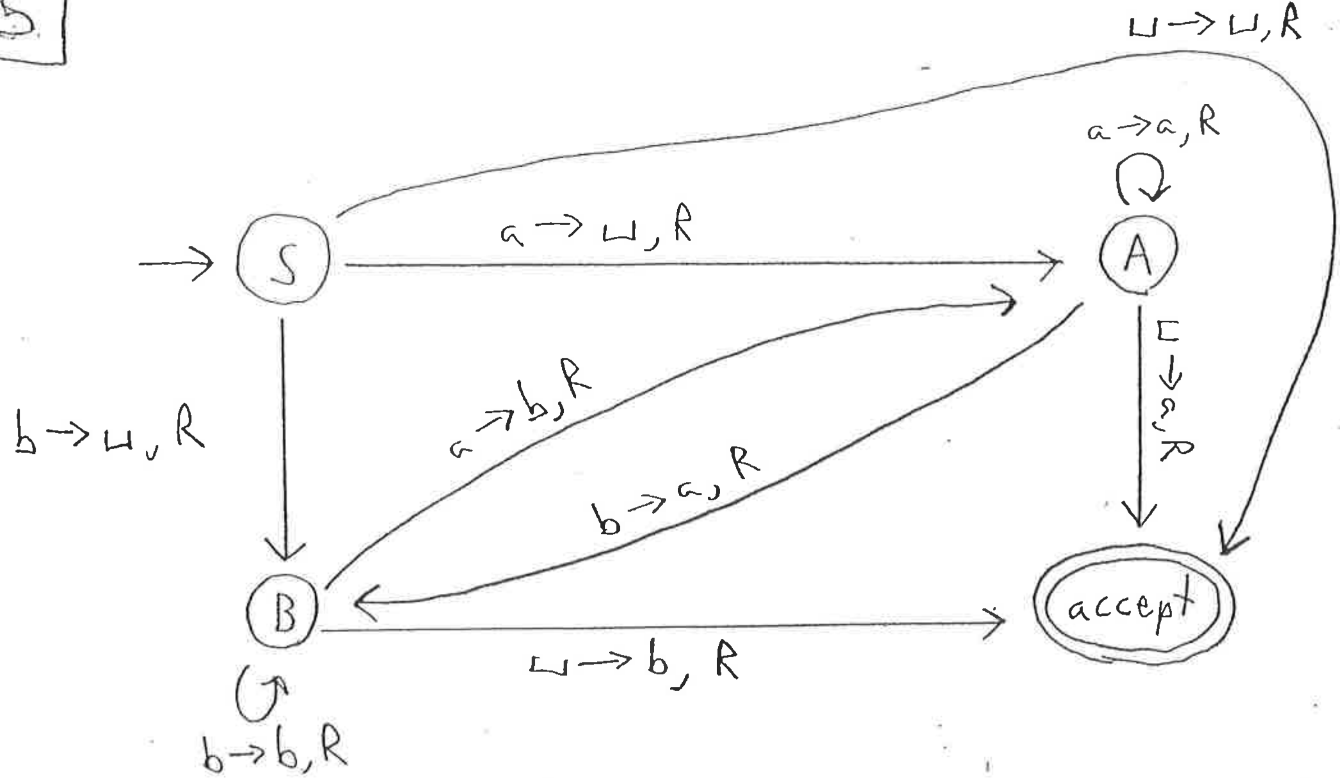

Explanation (not required). All transitions move tape head right.

	input	write	
Start state S	⊔	⊔	goto "accept"
	a	⊔	goto A
	b	⊔	goto B
A - state	a	a	goto A
	b	a	goto B
	⊔	a	goto "accept"
B - state	a	b	goto A
	b	b	goto B
	⊔	b	goto "accept"

Strictly, speaking there should be also a "reject" state, but there are no transitions to it.

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5. (sequence of configurations)

$S a a b a b$

$\sqcup A a b a b$

$\sqcup a A b a b$

$\sqcup a a B a b$

$\sqcup a a b A b$

$\sqcup a a b a B \sqcup$

$\sqcup a a b a b \sqcup q_{accept}$

6. M = "On input w:

1. Mark the first square of the tape by a primed symbol a' or b'. This is done so that the first square can be found later.

2. Make a left-to-right sweep until the first blank and replace the first two occurrences of "a" by X, and the first occurrence of b by X. The very first tape symbol (a' or b') is replaced by X'.

3. If two symbols "a" and one symbol "b" were found, return to the left end of the tape (that is marked by the unique primed symbol) and continue from 2.
If no symbols "a" and no symbols "b" were found, accept.
Otherwise, reject."