

**Computability and Complexity, CISC 462 - Assignment 2** (Fall 2018, K. Salomaa)  
**Due in lecture 10:30 AM, Thursday October 11**

1. Let  $M$  be the DFA depicted in Figure 1.

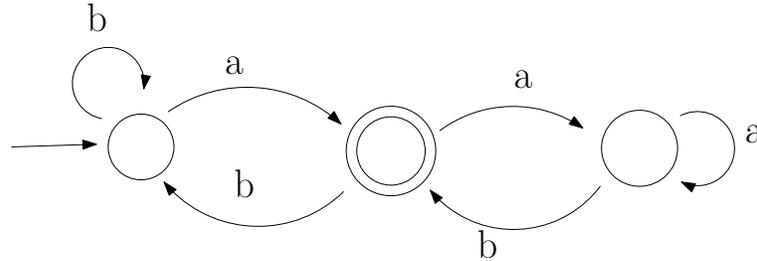


Figure 1: DFA  $M$  for question 1.

Answer the following questions and give reasons for your answers.

- (a) Is  $\langle M, bbaa \rangle \in A_{DFA}$ ?
- (b) Is  $\langle M, baaab \rangle \in A_{DFA}$ ?
- (c) Is  $\langle baab \rangle \in A_{DFA}$ ?
- (d) Is  $\langle M, a \rangle \in A_{REX}$ ? (The language  $A_{REX}$  is defined in early part of chapter 4.)
- (e) Is  $\langle M \rangle \in E_{DFA}$ ?
- (f) Is  $\langle M \rangle \in EQ_{DFA}$ ?
- (g) Is  $\langle M, M \rangle \in EQ_{DFA}$ ?

2. Consider the problem of deciding whether an NFA accepts all strings. Denote

$$ALL_{NFA} = \{ \langle M \rangle \mid M \text{ is an NFA and } L(M) = \Sigma^* \}.$$

Show that  $ALL_{NFA}$  is decidable.

3. (a) (3 marks) Consider the function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $f(n) = n^2 - 2n + 1$ . Answer the following questions and give reasons for your answers.
- Is  $f$  onto?
  - Is  $f$  one-to-one?
  - Is  $f$  a correspondence?
- (b) (7 marks) Show that the set of finite subsets of  $\mathbb{N}$  is countable.

4. Define

$$A = \{ \langle R \rangle \mid R \text{ is a regular expression describing a language } L(R) \text{ where all strings of } L(R) \text{ have } 0000 \text{ or } 111 \text{ as a substring} \}.$$

Show that  $A$  is decidable.

5. Define

$$B = \{ \langle N, u \rangle \mid N \text{ is a TM, } u \text{ is a string and } N \text{ accepts } u^p \text{ for some positive integer } p \}.$$

**Without using** Rice's theorem show that  $B$  is undecidable. That is, you need to explicitly reduce some known undecidable language to  $B$ .

6. A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states.
- Formulate this problem as a language and show that it is undecidable.

#### Regulations on Assignments

- As described on the course homepage, all assignments must be based on *individual work*.
- The assignments are graded according to the correctness, preciseness and elegance of the solutions.
- If, as part of your solution, you rely on results from the textbook you should clearly state which result(s) you are using.
- Each question is worth 10 marks and the assignment is marked out of 60 marks.