CISC 462 ASN 3 SOLUTIONS

$\boxed{1.}$ a) $\left[\frac{00}{001}\right]\left[\frac{11}{10}\right]\left[\frac{00}{001}\right]\left[\frac{011}{1}\right]$

b) i) Match: $\left[\frac{1}{101}\right]$ $\left[\frac{011}{1}\right]$

ii) This instance has the following property for all dominos $\left[\frac{x}{y}\right]$:

x is not a suffix of y and y is not a suffix of x.

The instance has <u>no</u> match because no domino could end a match.

Consider PCP instance $(u_1, v_1), \ldots, (u_m, v_m)$

$\boxed{2.}$ Define

$L_u = \{u_1, \ldots, u_m\}^+$, $L_v = \{v_1, \ldots, v_m\}^+$

The instance has a non-constrained solution iff $L_u \cap L_v \neq \emptyset$.

Decider for L_{ncPCP}:

$Q =$ "On input $\langle (u_1, v_1), \ldots, (u_m, v_m) \rangle$

1. Using the regex-to-DFA conversion algorithm construct DFA A for L_u and DFA B for L_v.
2. Construct DFA C such that $L(C) = L(A) \cap L(B)$
3. Using decider for DFA-emptiness decide whether $L(C) = \emptyset$. If "yes" reject, and, if "no" accept"

3. a) Yes, $A = E_{CFG}$. $A \leq_m E_{CFG}$ via the identity function. E_{CFG} is decidable.

b) Yes, $B = E_{CFG}$. $E_{CFG} \leq_m B$ via the identity function.

c) No. E_{CFG} is decidable so, according to Theorem 5.22, $C \leq_m E_{CFG}$ implies that C is decidable.

d) Yes. $E_{CFG} \leq_m A_{TM}$ using following reduction f. Let M_1 be a TM that accepts string b and rejects bb. Define f by setting: (*)

$$f(\langle G \rangle) = \begin{cases} \langle M_1, b \rangle & \text{if } L(G) = \emptyset \\ \langle M_1, bb \rangle & \text{if } L(G) \neq \emptyset \end{cases}$$

f is a computable function because E_{CFG} is decidable.

f mapping reduces E_{CFG} to A_{TM} because

$$\begin{cases} L(G) = \emptyset \text{ implies } f(\langle G \rangle) \in A_{TM} \\ L(G) \neq \emptyset \text{ implies } f(\langle G \rangle) \notin A_{TM} \end{cases}$$

(*) If w does not encode a grammar, define $f(w) = \langle M_1, bb \rangle$.

4.

a) TRUE

b) FALSE

c) TRUE

d) TRUE

e) FALSE

f) NO (Explanation: $2^{n+5} = 32 \cdot 2^n$ so 2^n is not asymptotically less than 2^{n+5}.)

g) NO

h) YES

i) YES

j) YES

$\boxed{5.}$ Decider for ALL_{DFA}:

Q = "On in ⟨A⟩ where A is a DFA

1. Using graph reachability algorithm find the set of reachable states of A.
2. If some reachable state is non-final or some reachable state has an undefined transition, reject. Otherwise accept"

The algorithm works because a DFA accepts all strings iff (all reachable states are final and all transitions from reachable states are defined).

The algorithm works in polynomial time because graph reachability is in P. (Theorem 7.14)

$\boxed{6.}$ a) Decider for TRIANGLE:

M = "On input $G=(V,E)$ where E contains unordered pairs of vertices:

1. For all $(x,y,z) \in V \times V \times V$:

   If $(x \neq y)$ & $(y \neq z)$ & $(x \neq z)$ & $(x,y) \in E$ & $(y,z) \in E$ & $(x,z) \in E$, then accept.

2. If did not accept up to this line, reject"

Time used:

The for-loop repeats $|V|^3$ times where $|V|$ is at most the length of input. Each iteration of the for-loop can be done by scanning through the list of edges, and checking equality of names of some vertices.

On a TM $\underbrace{\text{this}}_{\text{one iteration}}$ can be done in time $O(n^2)$ where n is the length of the input. (More precise: $O(n \cdot \log n)$.).

The total running time is $O(n^5)$.

$\boxed{6.}$ b) Using a similar algorithm for CLIQUE means that we have to go through k-tuples of elements of V where k is given as input. The for-loop repeats $|V|^k$ times and this is not polynomial as a function of the input length.