1. (a) Show that the polynomial time reducibility, \( \leq_P \), is a transitive relation.

(b) Prove the following:
   If \( A \in P \) then, for any language \( B \) that is not \( \emptyset \) or \( \Sigma^* \), we have \( A \leq_P B \).

2. Let \( \text{DOUBLE-SAT} = \{ \langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments} \} \).

   Show that \( \text{DOUBLE-SAT} \) is NP-complete. (You can assume known that “ordinary” satisfiability SAT is NP-complete.)

3. Let \( B \) be the language of properly nested parentheses. For example, \( (()) \) and \( (((()))()) \) are in \( B \), but \( )() \) is not. Show that \( B \) is in L (\( = \text{SPACE}(\log n) \)).

4. Define \( A_{\text{LBA}} = \{ \langle M, w \rangle \mid M \text{ is an LBA that accepts string } w \} \).

   Show that \( A_{\text{LBA}} \) is PSPACE-complete.

5. Recall that in Question 4 we showed that \( A_{\text{LBA}} \) is PSPACE-complete.

   (a) Is it known whether or not \( A_{\text{LBA}} \in \text{NL} \)? Explain your answer.

   (b) Is it known whether or not \( A_{\text{LBA}} \in P \)? Explain your answer.

6. What is the relationship (equal, strict inclusion in one direction (which one?), inclusion that is not known to be strict) between the following pairs of complexity classes. Justify your answers.

   (a) \( \text{NSPACE}(n \cdot \log n) \) and \( \text{SPACE}(n^3) \)

   (b) \( \text{TIME}(n^3 \cdot \log n) \) and \( \text{TIME}(n^3 \cdot \sqrt{n}) \)

   (c) \( \text{TIME}(2^n) \) and \( \text{TIME}(3^n) \)

   (d) \( \text{NTIME}(n \cdot \sqrt{n}) \) and \( \text{SPACE}(n^3 \cdot \log n) \)

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1 Here “known” means “known for sure” or that the claim has been proven.

2 Here “not known to be strict” means “on the basis of the results in chapters 7, 8 and 9 in our textbook, it is not known to be strict”. 
(e) $TIME(f(n))$ and $TIME(n^3)$ where

$$f(n) = \begin{cases} 
n^5, & \text{when } n \leq 2^{1000}, \\
n^3 \cdot \log n, & \text{otherwise.} \end{cases}$$

Regulations on Assignments

- As described on the course homepage, all assignments must be based on *individual work*.
- The assignments are graded according to the correctness, preciseness and elegance of the solutions.
- If, as part of your solution, you rely on results from the textbook you should clearly state which result(s) you are using.
- Each question is worth 10 marks and the assignment is marked out of 60 marks.