1. a) Match:
\[
\begin{array}{cccc}
\frac{ab}{abab} & \frac{ab}{abab} & \frac{ab\bar{c}}{b} & \frac{b}{a} \\
\frac{a\bar{a}}{a} & \frac{a\bar{a}}{a} & \frac{a\bar{a}}{a} & \frac{a\bar{a}}{a}
\end{array}
\]

b) i) Match: \[
\begin{array}{cccc}
\frac{aa}{aab} & \frac{bb}{aab} & \frac{aa}{aab} & \frac{a\bar{b}}{b}
\end{array}
\]

ii) All dominoes \[
\begin{array}{c}
\frac{x}{y}
\end{array}
\] in this instance have the following property:
\(x\) is not a suffix of \(y\) and \(y\) is not a suffix of \(x\).
The instance does not have a match because no domino could end a possible match.
2. a) False. Let $B = \{0^n1^n \mid n \geq 0\}$

Define $f: \Sigma^* \to \Sigma^*$ by setting

$$f(w) = \begin{cases} 01 \text{ if } w \in B \\ 10 \text{ if } w \notin B \end{cases}$$

$f$ is computable because $B$ is a decidable language.

$f$ mapping reduces $B$ to $0^*1^*$ but $B$ is non-regular.

b) True. Follows from Thm 5.22 because

$\{0^n1^n \mid n \geq 0\}$ is decidable.

(c) False. $A_{TM} \leq_m A_{TM}$ and $A_{TM}$ is undecidable.

3. a) TRUE  b) FALSE  c) FALSE  d) TRUE  e) TRUE  f) TRUE  
g) TRUE  h) FALSE  i) TRUE  j) FALSE
4. Suppose that TM $M_1$ decides $A$ in time $p_1(n)$ and TM $M_2$ decides $B$ in time $p_2(n)$ ($p_1, p_2$ polynomials).

i) Decide for $A \cup B$:

$P =$ "On input $w$:

1. Run $M_1$ on $w$. If $M_1$ accepts, then accept.

2. Run $M_2$ on $w$. If $M_2$ accepts, then accept. Else reject."  

The running time of $P$ is $p_1(n) + p_2(n)$.

ii) Decide for $A \cdot B$:

$Q =$ "On input $w = a_1 \cdots a_n$, $a_i \in \Sigma$, $i = 1, \ldots, n$

1. For $i = 0, \ldots, n$ do

   - Run $M_1$ on string $a_1 \cdots a_i$ and $M_2$ on string $a_{i+1} \cdots a_n$

   If both accept, then accept.

2. If hasn't accepted yet, then reject."
(ii) continued)

Running time of \( Q \):

Step 1. is repeated \( n+1 \) times, and each execution uses time less than \( p_1(n) + p_2(n) \).

The running time of \( Q \) is upper bounded by 
\[
(n+1) \left( p_1(n) + p_2(n) \right)
\]
which is a polynomial.

(iii) The language \( \Sigma^* - A \) is decided by \( \text{TM} \ M' \) that is obtained from \( M_1 \) by interchanging the accept and reject answers. The running time of \( M' \) is \( p_1(n) \).
\[ w = babab \]
We show that $B$ is decidable.

Denote $F = \{ \langle M \rangle \mid M$ is a single tape TM with input alphabet $\Sigma$, tape alphabet $\Gamma$ where the set of states is a subset of $\{0, 1, 2, \ldots, 99\}$ and $M$ halts on all inputs\}.

We say that a TM $M'$ is isomorphic to TM $M$ if $M'$ is obtained by a one-to-one renaming of the states.

Now $B$ can be decided by the following algorithm:

1. If $M$ has more than 100 states, reject. This can be checked easily from the description of $M$.

2. Check whether $M$ is isomorphic with some TM $N$ such that $\langle N \rangle \in F$. Note that $F$ is a finite set and, in particular, has only finitely many encodings of TMs that have the same number of states as $M$. Hence we can check whether $F$ has an isomorphic copy of $M$ using exhaustive search.

3. If $\langle N \rangle \in F$ such that $N$ is isomorphic to $M$ is found, accept. Otherwise, reject.
Why the above works:

- If \( \langle M \rangle \in B \), then clearly there exists \( M' \) isomorphic to \( M \) such that \( \langle M' \rangle \in F \). \( M' \) is obtained from \( M \) simply by renaming the states of \( M \) by symbols \( 0, 1, \ldots, i \) where \( i \leq 99 \).
- If \( \langle M \rangle \notin B \), then either
  
  (i) \( M \) has more than 100 states, or tape alphabet \( \neq \Gamma \),
  
  or
  
  or input alphabet \( \neq \Sigma \),

  (ii) \( M \) does not halt on some input.

In either case, \( M \) cannot be isomorphic with any \( M' \) such that \( \langle M' \rangle \in F \).

Note. We don't have an effective construction for the set \( F \). However, the algorithm with correctly chosen set \( F \) decides \( B \).