Queen's University, Faculty of Arts and Science, School of Computing
CISC-462 Final Exam, December XY, 2017
(Instructor: Kai Salomaa)

INSTRUCTIONS

- **Aids allowed:** You may bring in one 8.5×11 inch sheet of paper containing notes, and use it during the exam. The sheet can be written/printed on both sides.
- This examination is THREE HOURS in length.
- **Answer each question in the space provided** (on the question paper). There is an extra page at the end of the exam, if more space is needed. **Please write legibly.**
- Each of the 11 questions is worth 10 marks. The exam is marked out of 100 possible marks, and **your mark is calculated as the sum of your 10 best answers.** You can answer all 11 questions and only the 10 best answers are included in the mark.

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

STUDENT NUMBER: ______________________________

Please write extremely carefully.

Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Problem 5	/10
Problem 6	/10
Problem 7	/10
Problem 8	/10
Problem 9	/10
Problem 10	/10
Problem 11	/10
Total*	/100

*The total mark is calculated as the sum of your 10 best answers.

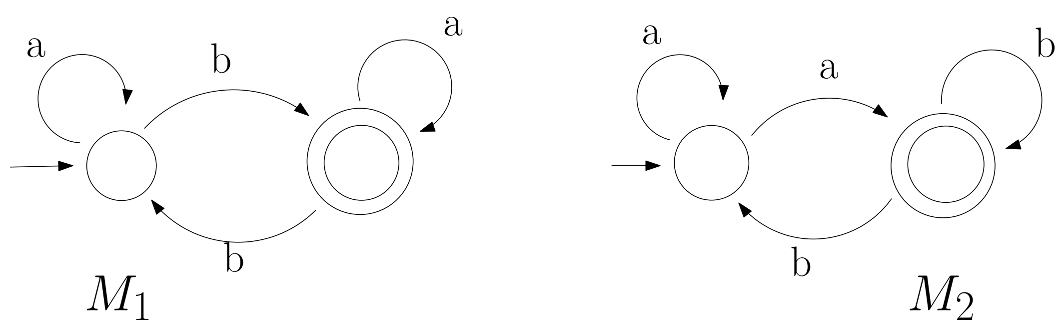

Figure 1: Finite automaton M_1 (on the left) and finite automaton M_2 (on the right).

1. Define:

$$A_{NFA} = \{ < M, w > \mid M \text{ is an NFA, } w \text{ is a string and } w \in L(M)\}$$

$$EQ_{DFA,REX} = \{ < M, R > \mid M \text{ is a DFA, } R \text{ is a regular expression, and } L(M) = L(R) \}.$$

Recall that DFA (respectively, NFA) stands for deterministic (respectively, nondeterministic) finite automaton. Let M_1 and M_2 be the finite automata given in Figure 1 and let R_1 be the regular expression $a^*b(a^*b)^*$

Answer the following questions and give <u>reasons</u> for your answers.

(i) Is $< M_1, aba > \in A_{NFA}$?

(ii) Is $< M_1, bab > \in A_{NFA}$?

(iii) Is $< M_2, abb > \in A_{NFA}$?

(iv) Is $< M_2, abab > \in A_{NFA}$?

(v) Is $< M_1, R_1 > \in EQ_{DFA,REX}$?

(vi) Is $< M_2, R_1 > \in EQ_{DFA,REX}$?

(vii) Is $< M_1, M_1 > \in EQ_{DFA,REX}$?

(viii) Is $< M_1, M_2 > \in EQ_{DFA,REX}$?

(ix) Is $< R_1, R_1 > \in EQ_{DFA,REX}$?

2. Give a complete construction, that is, a state transition diagram of a **single-tape deterministic Turing machine** M that decides the language

$$\{ \ b^i c^i \ \mid \ i \geq 0 \ \}$$

- Give also the sequence of configurations that your Turing machine M enters when started on input string bbc. (Note: The string bbc should be rejected.)

3. (i) (3 marks) Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = (2 \cdot n) - 1$. (Here $\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of positive integers.) Answer the following questions and give reasons for your answers.

   (a) Is f onto?

   (b) Is f one-to-one?

   (c) Is f a correspondence?

   (ii) (7 marks) Let $T = \{(i, j, k) \mid i, j, k \in \mathbb{N}\}$. Show that T is countable.

4. Give an implementation-level description of a **deterministic** Turing machine M that decides the following language A over the alphabet $\Sigma = \{b, c\}$.

$$A = \{\ w \in \Sigma^* \ \mid\ |w|_b = 2^{(|w|_c)}\ \}$$

Above the number of occurrences of symbol b (respectively, c) in a string w is denoted as $|w|_b$ (respectively, $|w|_c$). "$2^{(|w|_c)}$" denotes "2 to the power $|w|_c$".
If you wish, your deterministic TM M can use more than one tape.

5. (i) (4 marks) Does the following instance of the Post Correspondence Problem have a match (that is, a solution). Justify your answer.

$$\{ \ [\frac{baab}{ab}], \ [\frac{bab}{baab}], \ [\frac{baab}{baaab}], \ [\frac{ab}{abba}] \ \}$$

(ii) (6 marks) We define the following language

$PCP = \{< P > \mid \ P$ is an instance of the Post Correspondence Problem with a match $\}$

Answer the following questions and **justify** your answers.

(a) Does there exist a decidable language A such that $PCP \leq_m A$?

(b) Does there exist an undecidable language B such that $PCP \leq_m B$?

(c) Does there exist a decidable language C such that $C \leq_m PCP$?

6. Let

$$\mathrm{B}_{\mathrm{TM}} = \{ < M > \quad | \quad M \text{ is a deterministic Turing machine with input alphabet } \{a, b\}, \text{ such that all strings accepted by } M \text{ begin with } b \text{ and } L(M) \neq \emptyset \ \}.$$

Without using Rice's theorem show that B_{TM} is *undecidable*.

7. (i) (4 marks) In each part circle the correct answer.

(a) $n^2 = O((\log n)^5 \cdot n)$ TRUE FALSE

(b) $n \cdot \log n = o(n^2)$ TRUE FALSE

(c) $2^n = o(3^n)$ TRUE FALSE

(d) $1 = O(n^2)$ TRUE FALSE

(ii) (6 marks) We define

$$\text{ALL}_{DFA} = \{ < A > \mid A \text{ is a DFA with alphabet } \Sigma \text{ and } L(A) = \Sigma^* \}.$$

<u>Show</u> that ALL_{DFA} is in P. In your solution you can assume known that the reachability problem on directed graphs (PATH) is in P.

8. Give an example of a non-context-free language A such that A is in the class L (= SPACE($\log n$)). You should briefly explain how a logarithmic space deterministic TM decides your language A. You do not need to prove that A is non-context-free.

9. Let B be NP-complete and assume that $B \leq_P \overline{B}$. (Here $\overline{B}$ is the complement of B and $\leq_P$ is polynomial time reducibility.)

   Show that the above implies NP = coNP.

10. Recall that $\leq_L$ denotes the logarithmic space reducibility. Answer the following questions and justify your answers. *Note:* To show that an implication does not hold, one should give a counter-example.

    (i) Let B be an NP-complete language and A is a language such that $A \leq_L B$. Does this imply that A is in NP ?

    (ii) Let B be a context-free language and A is a language such that $A \leq_L B$. Does this imply that A is in NP ?

    (iii) Let B be a context-free language and A is a language such that $B \leq_L A$. Does this imply that A is in NP ?

11. What is the relationship (equal "$=$"; strict inclusion "$\subset$" or "$\supset$"; inclusion that is not known to be strict "$\subseteq$" or "$\supseteq$") between the following pairs of complexity classes. **Justify** your answers.

    (i) $\mathrm{TIME}(n^3)$ and $\mathrm{TIME}(n^3 \cdot \log n)$

    (ii) $\mathrm{SPACE}(n^3 + n^2 \cdot (\log n)^3)$ and $\mathrm{SPACE}(n^3 \cdot \log n)$

    (iii) $\mathrm{SPACE}(2^n)$ and $\mathrm{SPACE}(2^{n+5})$

    (iv) $\mathrm{TIME}(2^n)$ and $\mathrm{TIME}(3^n)$

    (v) $\mathrm{SPACE}(n^5)$ and $\mathrm{NSPACE}(n^2 \cdot \log n)$

Extra page.