School of Computing, Queen's University

CISC-462 Practice Midterm, fall 2018

INSTRUCTIONS

- You have 50 minutes time. This is a closed book test. You can bring with you one standard size (8.5 × 11 inch) sheet of notes and use it during the test. The sheet can be written on both sides.
- **Answer each question in the space provided** (on the question paper). There is an extra page at the end of the exam if more space is needed. **Please write legibly.**
- Each question is worth 10 marks. The exam is marked out of 40 possible marks and your mark is calculated as the sum of your 4 best answers. If you wish, you can answer all five questions, however, only the 4 best answers are used to compute your mark.

NAME (optional): _____

STUDENT NUMBER: _____ (REQUIRED)

MARKS

Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Problem 5	/10
Total*:	/40

*The total mark is calculated as the sum of your 4 best answers.

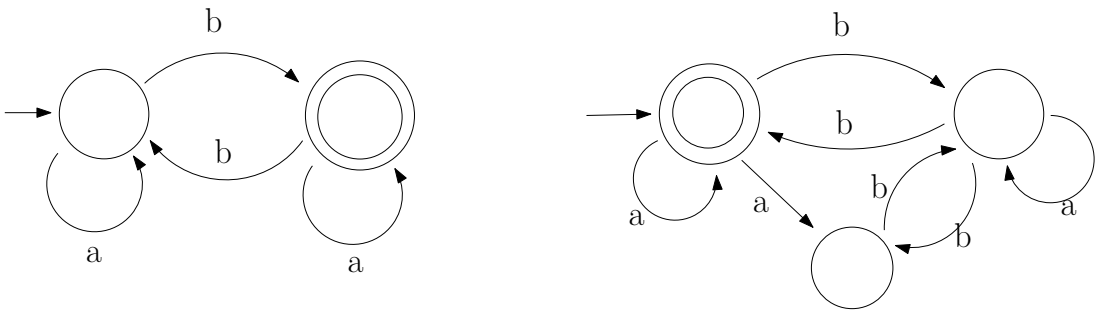

Figure 1: A finite automaton M_1 (on left) and a finite automaton M_2 (on right).

1. Define:

$$A_{NFA} = \{ < B, w > \mid B \text{ is an NFA that accepts input string } w\},$$

$$\text{EQ}_{DFA} = \{ < B, C > \mid B, C \text{ are DFAs and } L(B) = L(C) \}.$$

Recall that DFA (respectively, NFA) stands for deterministic (respectively, nondeterministic) finite automaton. Answer the following questions (for M_1 and M_2 given in the above figure) and give reasons for your asnwers.

(a) Is $< M_1, babb > \in A_{NFA}$?

(b) Is $< M_1, abba > \in A_{NFA}$?

(c) Is $< M_2, abb > \in A_{NFA}$?

(d) Is $< M_2, aaabb > \in A_{NFA}$?

(e) Is $< M_1, M_2 > \in \text{EQ}_{DFA}$?

(f) Is $< M_1, M_1 > \in \text{EQ}_{DFA}$?

(g) Is $< M_2, M_2 > \in \text{EQ}_{DFA}$?

2. (a) Consider the language

$C = \{< M, w > \mid M$ is a DFA, and some string of $L(M)$ contains string w as a substring $\}$.

Is the language C decidable or undecidable? Prove your answer.

(b) Consider the language

$D = \{< M > \mid M$ is a Turing machine and there exists a DFA A such that $L(M) = L(A)\}$.

Is the language D decidable or undecidable? Prove your answer.

3. Give an implementation-level description of a **deterministic** one tape Turing machine that decides the following language A over the alphabet $\Sigma = \{c, d\}$. The number of occurrences of symbol c (respectively, d) in a string w is denoted $|w|_c$ (respectively, $|w|_d$).

$$A = \{\ w \in \Sigma^* \ \mid \ |w|_c \leq |w|_d \leq 2 \cdot |w|_c \ \}.$$

That is, in strings of A the number of occurrences of d is at least the number of occurrences of c and at most 2 times the number of occurrences of c.

4. Let

$$\text{TWO}_{\text{TM}} = \{ < M > \quad | \quad M \text{ is a deterministic Turing machine and } L(M) \text{ consists of exactly two strings } \}.$$

Without using Rice's theorem show that TWO_{TM} is *undecidable*.

5. (a) (7 marks) Let $T = \{ (i, k, m) \mid i, k, m \in \mathbb{N} \}$.
Show that the set T is countable.

(b) (3 marks) Are the following sets countable? For each case circle the correct answer – no explanation needed. If you circle both YES and NO, it is considered a wrong answer.

- The set $\{0, 1\}^*$. YES NO
- The set of all subsets of $\{0, 1\}^*$. YES NO
- The set of all finite subsets of $\{0, 1\}^*$. YES NO

(Extra page)