INSTRUCTIONS

• You have 50 minutes time. This is a closed book test. You can bring with you one standard size (8.5 × 11 inch) sheet of notes and use it during the test. The sheet can be written on both sides.

• Answer each question in the space provided (on the question paper). There is an extra page at the end of the exam if more space is needed. Please write legibly.

• Each question is worth 10 marks. The exam is marked out of 40 possible marks and your mark is calculated as the sum of your 4 best answers. If you wish, you can answer all five questions, however, only the 4 best answers are used to compute your mark.

NAME (optional): ________________________________

STUDENT NUMBER: __________________________ (REQUIRED)

MARKS

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*The total mark is calculated as the sum of your 4 best answers.
1. Define:

\[ A_{NFA} = \{ < B, w > \mid B \text{ is an NFA that accepts input string } w \} \]

\[ EQ_{DFA} = \{ < B, C > \mid B, C \text{ are DFAs and } L(B) = L(C) \} \]

Recall that DFA (respectively, NFA) stands for deterministic (respectively, nondeterministic) finite automaton. Answer the following questions (for \( M_1 \) and \( M_2 \) given in the above figure) and give reasons for your answers.

(a) Is \( < M_1, babb > \in A_{NFA} \)?

(b) Is \( < M_1, abba > \in A_{NFA} \)?

(c) Is \( < M_2, abb > \in A_{NFA} \)?

(d) Is \( < M_2, aaabb > \in A_{NFA} \)?

(e) Is \( < M_1, M_2 > \in EQ_{DFA} \)?

(f) Is \( < M_1, M_1 > \in EQ_{DFA} \)?

(g) Is \( < M_2, M_2 > \in EQ_{DFA} \)?
2. (a) Consider the language

\[ C = \{ < M, w > \mid M \text{ is a DFA, and some string of } L(M) \text{ contains string } w \text{ as a substring } \} \].

Is the language \( C \) decidable or undecidable? Prove your answer.

(b) Consider the language

\[ D = \{ < M > \mid M \text{ is a Turing machine and there exists a DFA } A \text{ such that } L(M) = L(A) \} \].

Is the language \( D \) decidable or undecidable? Prove your answer.
3. Give an implementation-level description of a deterministic one tape Turing machine that decides the following language $A$ over the alphabet $\Sigma = \{c, d\}$. The number of occurrences of symbol $c$ (respectively, $d$) in a string $w$ is denoted $|w|_c$ (respectively, $|w|_d$).

$$A = \{ w \in \Sigma^* \mid |w|_c \leq |w|_d \leq 2 \cdot |w|_c \}.$$ 

That is, in strings of $A$ the number of occurrences of $d$ is at least the number of occurrences of $c$ and at most 2 times the number of occurrences of $c$. 
4. Let

\[ \text{TWO}_{\text{TM}} = \{ < M > \mid M \text{ is a deterministic Turing machine and } L(M) \text{ consists of exactly two strings} \}. \]

**Without using** Rice’s theorem show that \( \text{TWO}_{\text{TM}} \) is *undecidable*. 

5. (a) (7 marks) Let \( T = \{ (i, k, m) \mid i, k, m \in \mathbb{N} \} \). Show that the set \( T \) is countable.

(b) (3 marks) Are the following sets countable? For each case circle the correct answer – no explanation needed. If you circle both YES and NO, it is considered a wrong answer.

- The set \( \{0, 1\}^* \). YES NO
- The set of all subsets of \( \{0, 1\}^* \). YES NO
- The set of all finite subsets of \( \{0, 1\}^* \). YES NO