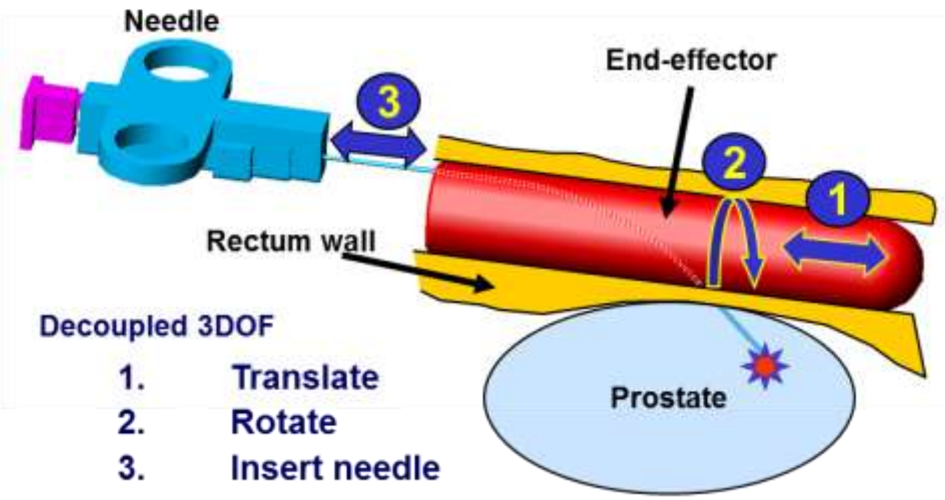
Transrectal Robot Navigation

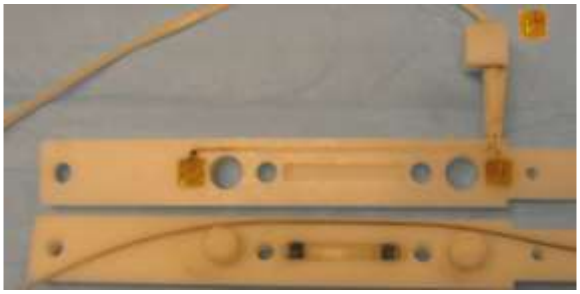
Problem Statement

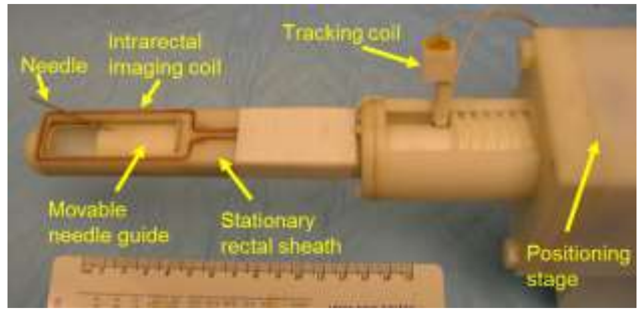
We will perform robot assisted transrectal prostate biopsy with MRI guidance with the robotic system shown below and discussed in class [Susil 2004, Krieger 2005].

The kinematic design of the device is shown below:



We instrumented the robot with 3 tracking antenna coils (a.k.a. markers) that report the position of the robot's end-effector in MRI scanner coordinates in real-time. The three tracking coils can be seen in the pictures below.



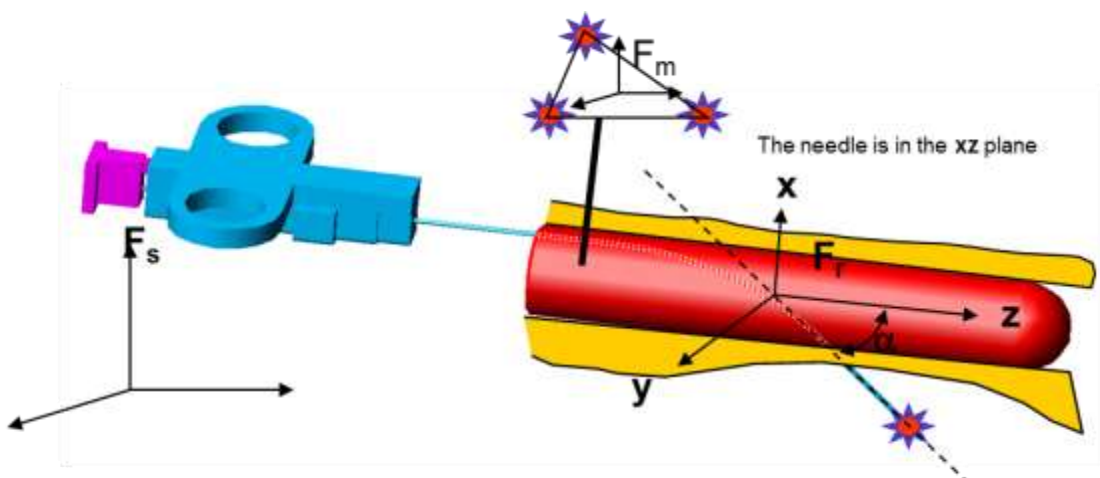


Calibration

Upon building the device, the position of the markers relative to the robot frame is not known. When we assemble the robot, the exact needle angle is not known and it must be measured experimentally as part of the calibration process. The coordinate system assigned to the robot's end-effector is marked in the figure as F_r. The coordinate frame transformation between marker frame and robot frame must be calibrated. For this purpose, we place a 4th marker in the needle tip. The tracking system reports the coordinates of the 4 markers (3 markers on the robot's end-effector and 1 marker on the needle tip) in MRI scanner frame.

During the calibration process, we place the robot in the MRI scanner inside the patient and then move the robot to its home position. The home position is mathematically arbitrary and is picked by convenience. To The home position should be picked as close as possible to the target space, so that the device will not move much during surgery from home position to any target position. This helps avoid collision and reduce unwanted patient motion.

According to the kinematics of the device, we can perform combinations of the following motions: rotate about z axis, translate along the z axis, and insert/retract the needle. In each position we read the positions of the four markers in MRI scanner coordinates. From these recordings we compute the object of calibration: the frame transformation that transforms an arbitrary point from scanner frame to robot frame.



Q1: Mark the relevant coordinate frame transformations in the sketch above and write up the formula for the transformation of a target point from F_s scanner frame to F_r robot frame through F_m marker frame. (**5pts**)

Q2: Explain, step by step, how you compute F_m marker frame from the three markers (M_1, M_2, M_3) (**5pts**)

Q3: Compose the necessary sequence of calibration motions that allow you to compute the F_r robot frame and needle angle. A coordinate frame is defined by three mutually ortho-normal base vectors and a center of the coordinate system. Therefore you must calculate each base vector and the center of the coordinate system. Explain your process and derive the mathematics and produce the necessary chain of formulas that will lead to (1) the base vectors, (2) origin of the robot frame, and (3) the exit angle of the needle. (**20pts**)

Q4: Explain how to make each of the three calibration processes more robust to tracking errors (**5pts**).

Q5: Using the calibration parameters obtained above derive the 4x4 homogeneous frame transformation matrix that transforms an arbitrary point from F_s MRI scanner fame to Fr robot frame. For simplicity, you can assume that F_s is in canonical position with vectors (1,0,0), (0,1,0), (0,0,1) (**5pts**)

Workspace Analysis

During design of the device, it is critical to ascertain that all relevant target points can be safely reached with the robot. We need to compute the minimum range of motion for each degree of freedom and make sure the robot can perform those. For this question, assume the following:

- The diameter of the robot's cylindrical end effector 30mm
- The needle exit angle 45 degree
- The shape of the prostate is spherical. 40mm diameter
- The thickness of the rectum wall is uniform 5mm

Q6: **Work space**: Derive math formulas to compute the minimum required range of motion for each degree of freedom of the robot, in order to be able to sample the largest prostate. Make a sketch for each degree of freedom. (**10pts**)

Kinematics

During surgery, we first move the robot to its home position. We receive the coordinates of the 3 markers and the intended target point in MRI scanner coordinates. Having calibrated the robot, we transform the intended target point from MRI scanner fame to robot frame. Here we encounter an issue called ***inverse and forward kinematics***: we need to compute the sequence of motion for the robot to reach this target point. For this particular 3-DOF robot, we need the following: (1) translation of the robot's end effector in the rectum, (2) rotation of the robot's end effector inside the rectum, (3) insertion depth of the needle.

Q7: **Forward kinematics**: In order to use the robot in surgery, we need to be able to compute the resulting location of the needle tip upon moving the motion stages of the robot from its home position. This is called forward kinematics. Derive the necessary math formulas to compute the location of the needle tip in F_r robot coordinate frame as a function of the translation, rotation and needle insertion relative to the home position of the robot. (**15pts**)

Q8: **Inverse kinematics**: Derive the necessary math formulas to compute the values of translation, rotation and needle insertion that take the needle tip to a given target point from the home position of the device. Derive the mathematics and produce the necessary chain of formulas that will yield the three parameters. (**15pts**)

Q9: Implement the forward kinematics in MATLAB (**5pts**)

Q9: Implement the inverse kinematics in MATLAB (**5pts**)

Q10: To test the forward and inverse kinematics, generate a handful of special ground truth examples of which you know the solution up front. For this test, use the assumptions made in Workspace Analysis.

Test 3 simple ground truth examples for simple decoupled motions

Generate N=10 or so kinematics sequences of random motions within the Workspace. Demonstrate that your code is correct, by first performing random forward motions and then inverting each motion to receive back the original forward motion parameters. (**10pts**)

Directions

- In answering each question, explain your process step by step. You should use figures, sketches, flow charts, pseudo code, or anything you need to convey your thoughts in a clear and concise manner.
- You must demonstrate that you understand what you are doing and I must be able to follow your reasoning. Depending on the quality your reasoning and depth of discussion of results, you may gain (or lose) lots of points.
- Read the online syllabus carefully for general instructions on the submission of assignments.
- **Have fun!**