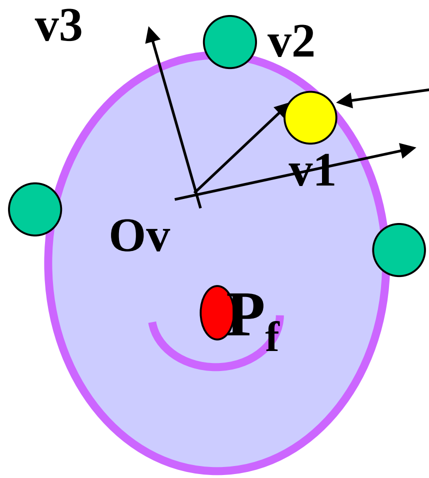
More on Transformations: Point sets

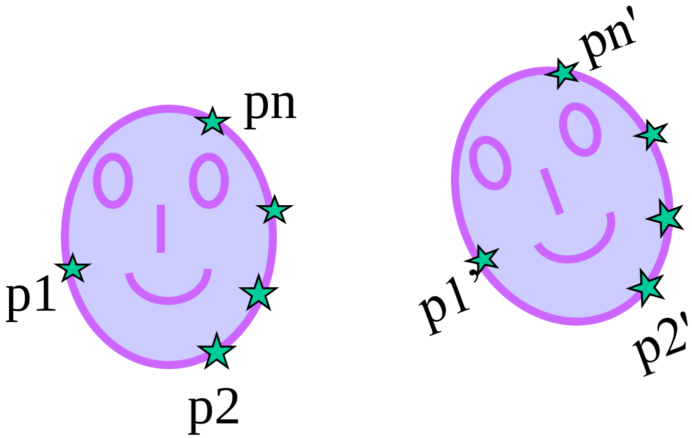
Redundant (N>3) fiducials



Add more fiducials

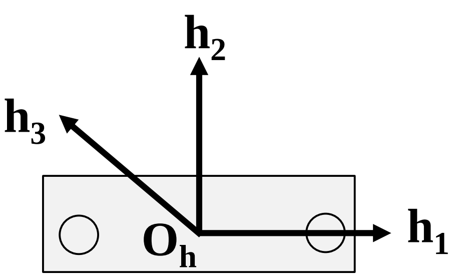
- The constrain the registration more strongly.
- But they must be placed distributed wisely (same applies as before!)
- We no longer simply assign an orthonomal frame to the body, we cannot (easily) use our method based on 3 fiducials.
- We can use redundant fiducial to eliminate fiducials with large FLE
- We can select the “most congruent” pair of triangles for registration
- Can we use the available fiducials?

Multiple observations of a moving body



{p} {p'}

- A rigid body (such as the head) is moving during surgery
- {p1, p2, p3…} markers were affixed to the rigid head
- The moving markers are continuously observed (such as with optical stereo camera) as {p1', p2', p3}
- We want to know the transformation that takes an arbitrary point of the head from one pose to another



Observation (home) coordinate system

Typically in the home: $\mathbf{O_h} = (0,0,0)$, $\mathbf{h_1} = (1,0,0)$, $\mathbf{h_2} = (0,1,0)$, $\mathbf{h_3} = (0,0,1)$

Least-Squares Fitting of Two 3-D Point Sets

K. S. ARUN, T. S. HUANG, AND S. D. BLOSTEIN,1987

In many computer vision applications, notably the estimation of motion parameters of a rigid object using 3-D point correspondences [1] and the determination of the relative attitude of a rigid object with respect to a reference [2], we encounter the following mathematical problem. We are given two 3-D point sets $\{p_i\}$; $i = 1, 2, \cdots, N$ (here, p_i and p_i' are considered as 3×1 column matrices)

$$p_i' = Rp_i + T + N_i \tag{1}$$

where R is a 3×3 rotation matrix, T is a translation vector (3×1 column matrix), and N_i a noise vector. (We assume that the rotation is around an axis passing through the origin). We want to find R and T to minimize

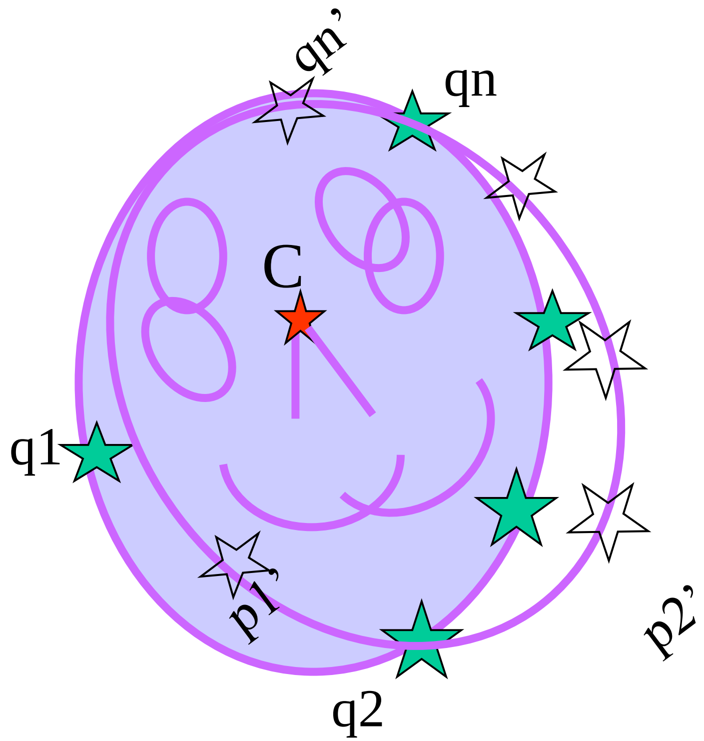
$$\Sigma^2 = \sum_{i=1}^{N} \left\| p_i' - (Rp_i + T) \right\|^2. \tag{2}$$

Decouple translation and rotation

It can be shown that in the least square solution of this, Trans and Rot can be decupled, and the optimum translation is in the Centroid(C) (center of gravity, aka average of all markers.)

Then we have

$$\Sigma^2 = \sum_{i=1}^{N} \| q_i' - Rq_i \|^2.$$



SVD algorithm for finding R

Algorithm

Step 1: From $\{p_i\}$, $\{p'_i\}$ calculate p, p'; and then $\{q_i\}$, $\{q'_i\}$.

Step 2: Calculate the 3×3 matrix

$$H \triangleq \sum_{i=1}^{N} q_i q_i'^t \tag{11}$$

where the superscript t denotes matrix transposition.

Step 3: Find the SVD of H,

$$H = U \Lambda V^t. \tag{12}$$

Step 4: Calculate

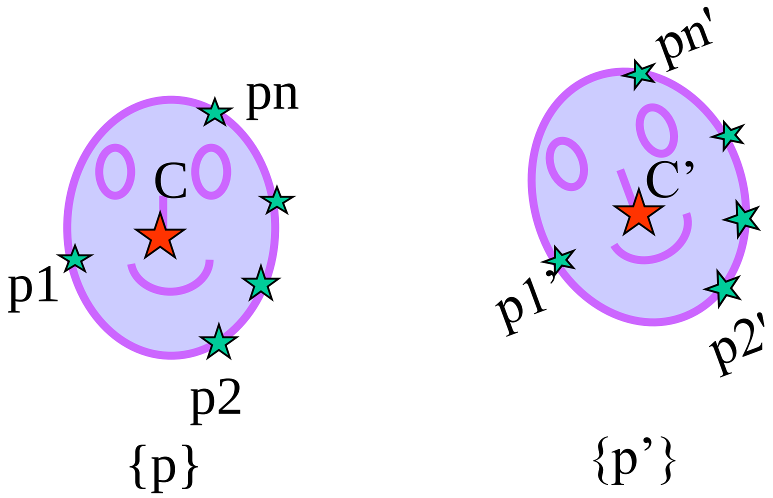
$$X = VU^t. \tag{13}$$

Step 5: Calculate, det (x), the determinant of X.

If det $(x) = +1$, then $\hat{R} = X$.

If det $(x) = -1$, the algorithm fails. (This case usually does not occur. See Sections IV and V.)

Simple alternative to Arun's



Compute the centre of gravity in C={p} and C'={p'}

For each possible triplet of makers:

- Create the usual e base in {p} and v base in {p'}
- Always place the e base in C a the v base in C'
- Compute $F_{v\leftarrow e} = T_{v\leftarrow h} R_{v\leftarrow h} R_{h\leftarrow e} T_{h\leftarrow e}$
- Simply average the resulting $F_{v\leftarrow e}$ transforms from each triplet
- (Note that this will yield the same averaging $R_{v\leftarrow h2}$ $R_{h1\leftarrow e}$)

Not numerically optimal, but close

Not a "proper" transform matrix → NOT invertible!

To get the inverse transform, reverse the above process: compute all possible $F_{h1\leftarrow h2}$ and average them.