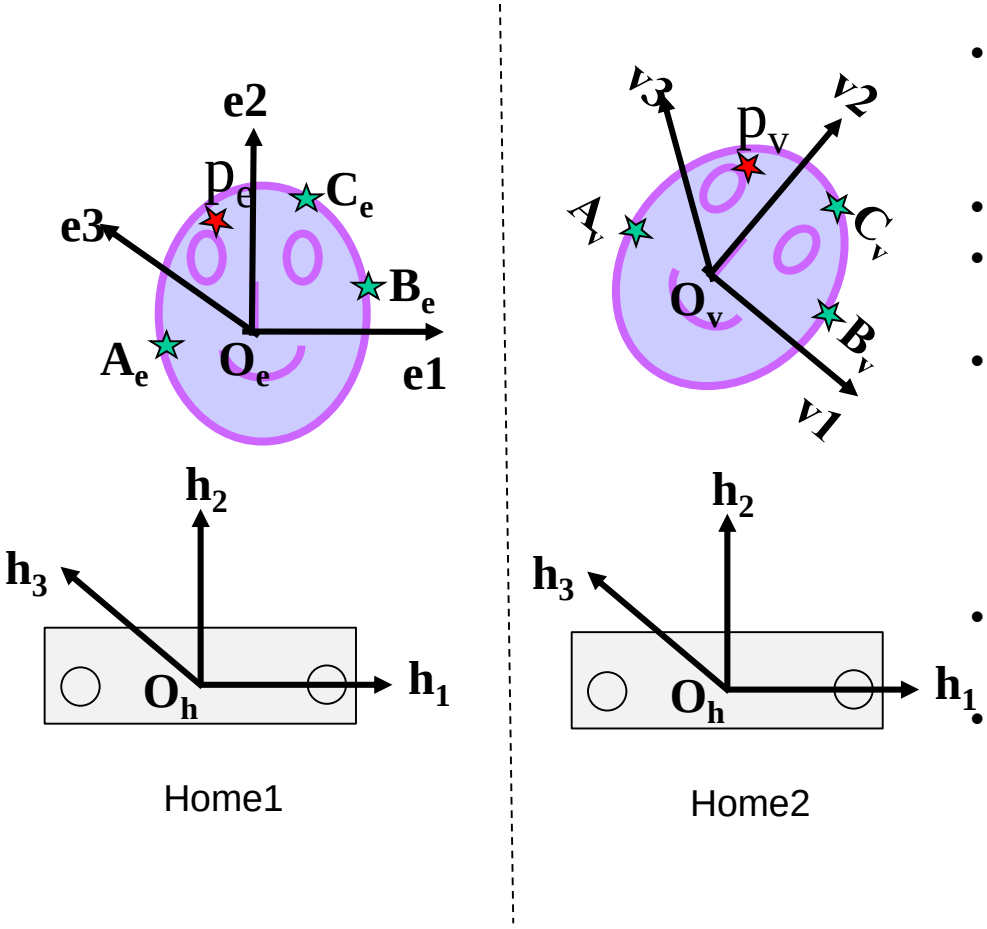
More on Transformations: FLE, FRE and TRE

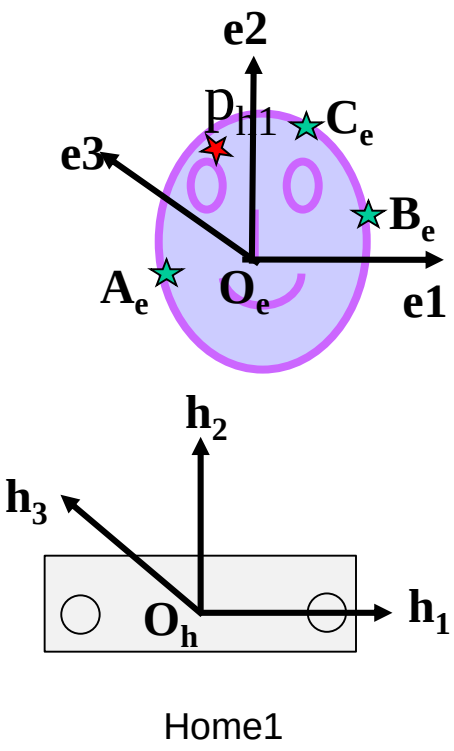
Transform a body between two homes

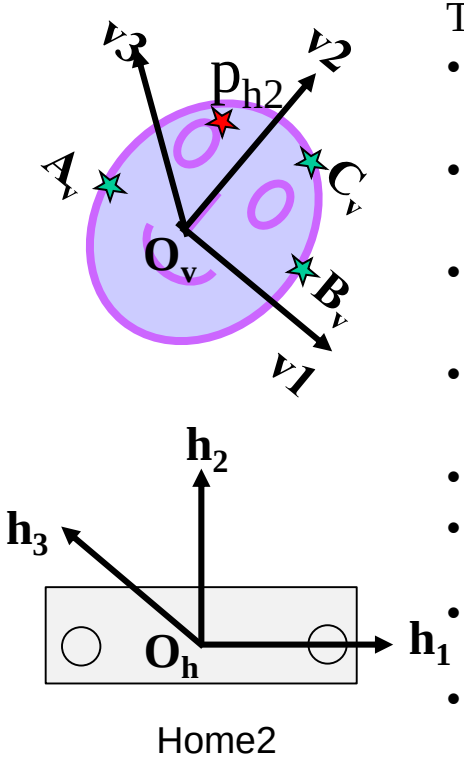


- A rigid body (such as the head) is observed in two home frames (such as two different imaging modalities at different times)
- ABC markers are affixed to the rigid head
- The markers are localized in each home frame
- We want to know the transformation that takes an arbitrary point of the head from one home to another. (Say, because we want to merge different imaging modalities of the same head.)
- This transformation will be expressed as a series of translations and rotations…
- In each home, we compute the frame defined by the ABC markers:
  - e_1,e_2,e_3,O_e
  - v_1,v_2,v_3,O_v

Typically in each home: $\mathbf{O_h} = (0,0,0)$, $\mathbf{h_1} = (1,0,0)$, $\mathbf{h_2} = (0,1,0)$, $\mathbf{h_3} = (0,0,1)$

Transform a body between two homes (cont'd)





Then we compute the following:

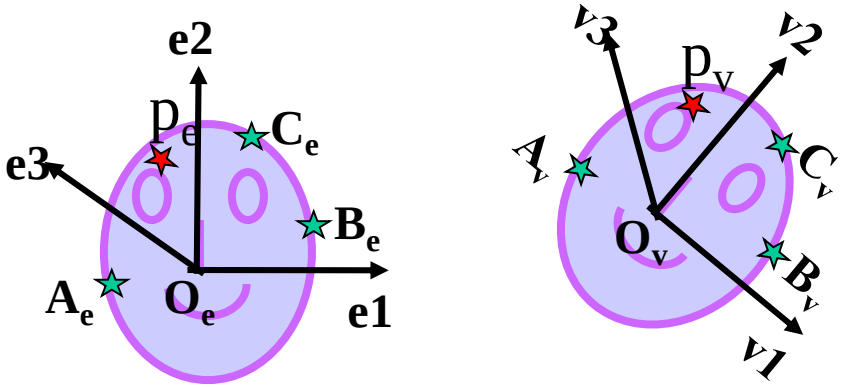
- $T_{h1\leftarrow e}$ translation that takes the O_e center of the e frame to the O_h center of the home1 frame
- $T_{h2\leftarrow v}$ translation that takes the O_v center of the v frame to the O_h center of the home2 frame.
- $R_{h1\leftarrow e}$ rotation that rotates the e base vectors to home2 base vectors.
- $R_{h2\leftarrow v}$ rotation that rotates the v base vectors to home2 base vectors.
- Then we can write:
- $R_{h2\leftarrow v}\ T_{h2\leftarrow v}\ p_{h2} = R_{h1\leftarrow e}\ T_{h1\leftarrow e}\ p_{h1}$
- $p_{h2} = \underbrace{T_{v\leftarrow h2}\ R_{v\leftarrow h2}\ R_{h1\leftarrow e}\ T_{h1\leftarrow e}}_{F_{h2\leftarrow h1}}\ p_{h1}$
- $p_{h2} = F_{h2\leftarrow h1}\ p_{h1}$

Typically in each home: $\mathbf{O_h} = (0,0,0)$, $\mathbf{h_1} = (1,0,0)$, $\mathbf{h_2} = (0,1,0)$, $\mathbf{h_3} = (0,0,1)$

Fiducial Localization Error (FLE)

Fiducials must be localized in both ct abd mr
FLE – fiducial localization error
Why? – imager (resolution, distortion), image processing…

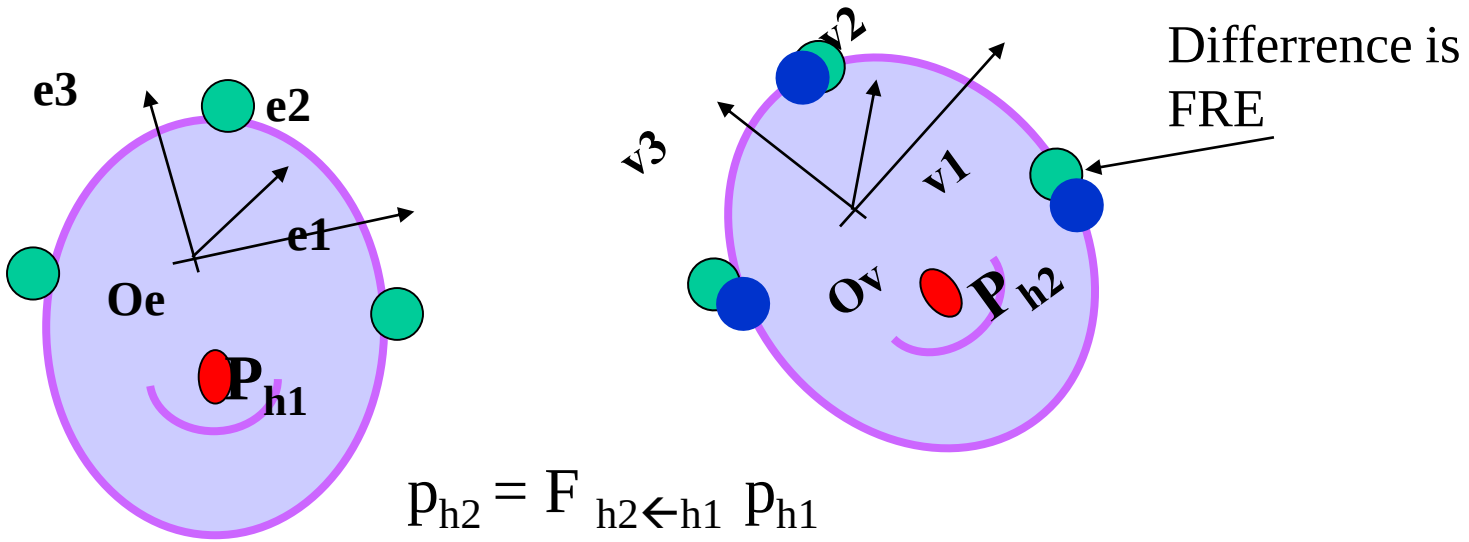
Detecting and measuring FLE



$$p_{h2} = F_{h2 \leftarrow h1}\ p_{h1}$$

- We cannot measure FLE
- But there are surrogate metrics, such as congruency
- Fiducials must define congruent sets
- Multiple possibilities for congruency metrics
- Does congruency check always detect FLE?

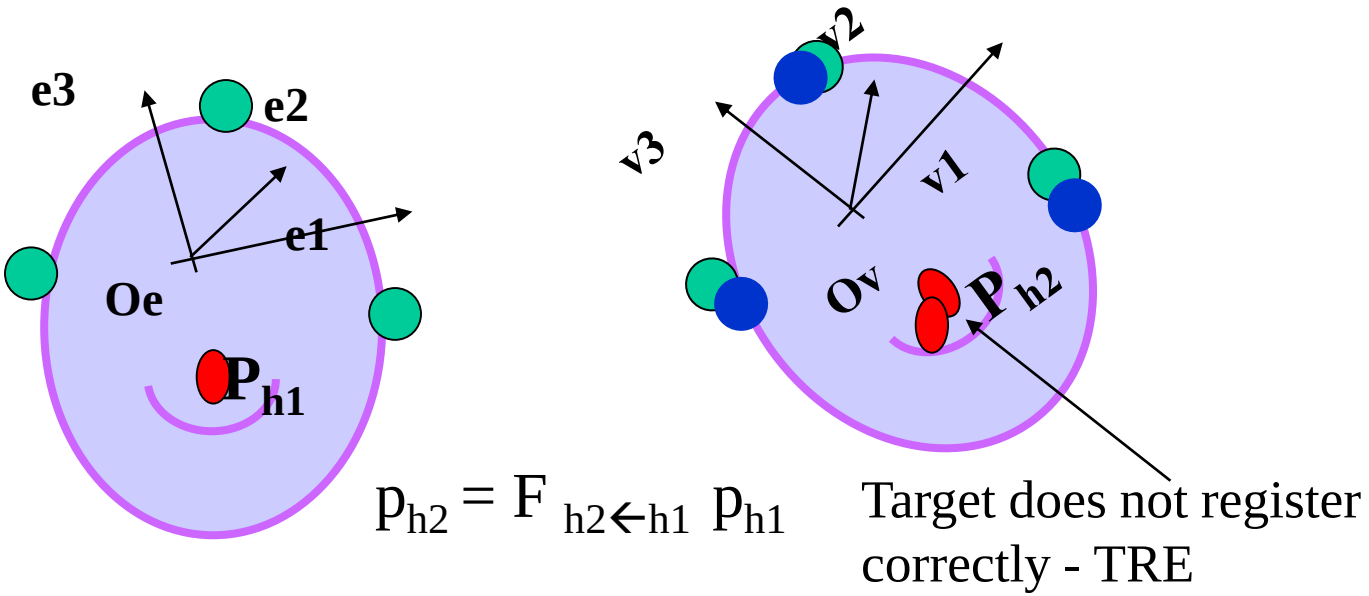
Fiducial Registration Error (FRE)



$$p_{h2} = F_{h2 \leftarrow h1}\ p_{h1}$$

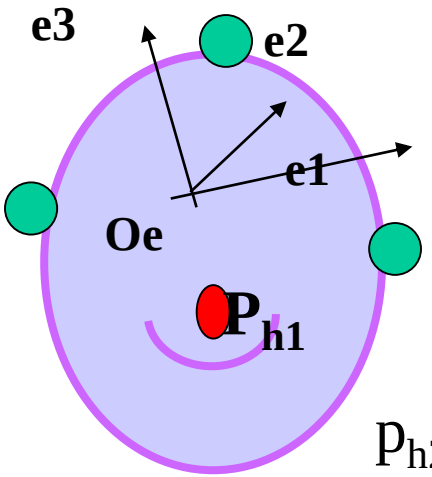
- FRE – fiducial registration error
  - We have computed the F transformation to take any point from h1 to h2.
  - Apply F on the fiducials. They will not cast exactly onto their counterparts
  - The difference is FRE

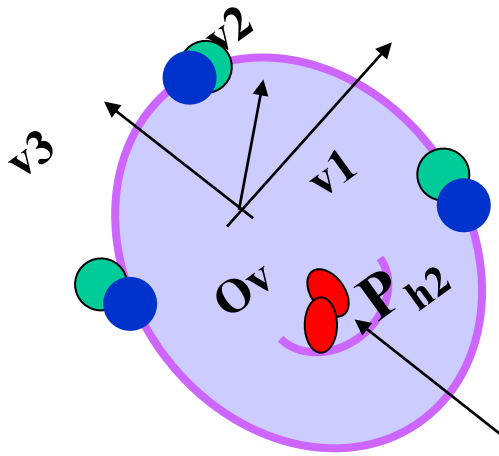
Target registration error (TRE)



- Target does not register correctly
- The difference is called TRE target registration error (TRE)
- Can we measure TRE during surgery?? We can't !
- But we can measure FRE, which is a strong indicator – but not a bullet-proof predictor

How to mitigate TRE?



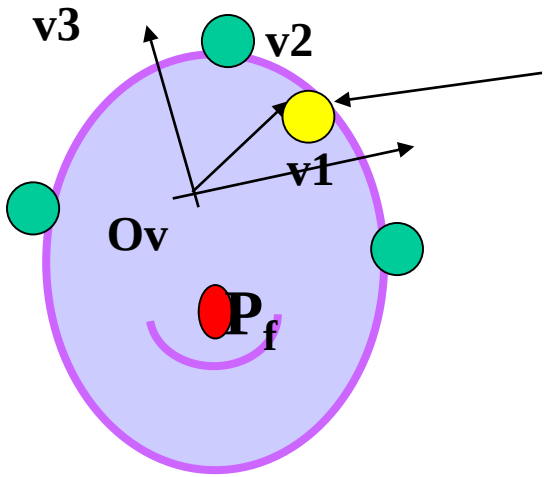


$p_{h2} = F_{h2 \leftarrow h1}\ p_{h1}$

Target does not register correctly - TRE

Reduce FLE – by perfect imaging and image processing :-)
Fiducials must move relative to target
Fiducials must move relative to one another
Select fiducial locations carefully to be resistant to rotation error
What happens if fiducials and target are in same plane?
Consider both in-plane and out-of-plane errors
Place fiducial so that target falls close to centroid

Redundant (N>3) fiducials



Add more fiducials

- The constrain the registration more strongly.
- But they must be placed distributed wisely (same applies as before!)
- We no longer simply assign an orthonomal frame to the body, we cannot (easily) use our method based on 3 fiducials.
- We can use redundant fiducial to eliminate fiducials with large FLE
- We can select the “most congruent” pair of triangles for registration
- How can we use ALL fiducials? … (next lecture)