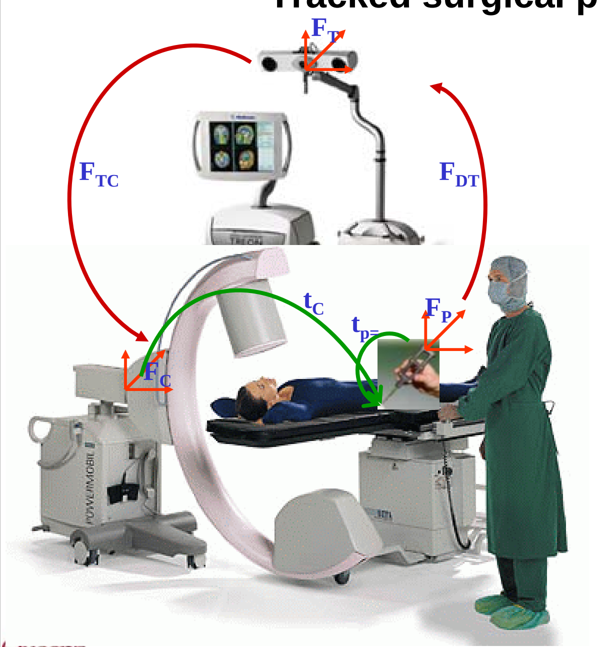
Calibration

Tracked surgical pointer

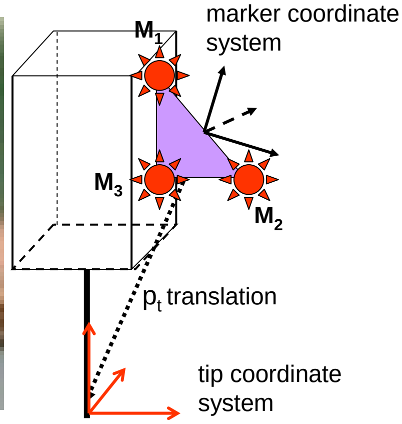


$p_{C=} (F_{TC}(F_{DT}\, t_p))$

t_p ???

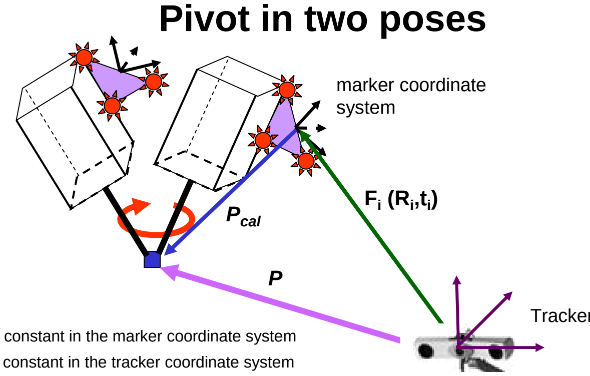
- Must be known prior to using the surgical tool
- Constant during the procedure
- We need a calibration session before surgery

Generic Pivot Calibration



- Determine p_t translation between tip and marker coordinate system
- Create a geometrical constraint -- pivot around a fixed point

Pivot in two poses



- p_{cal} vector is constant in the marker coordinate system
- Pivot point is constant in the tracker coordinate system
- M_1,M_2,M_3 are reported by the tracker in both poses
- Determine the marker coordinate system (orthonormal base from 3 markers) in both poses
- Calculate the F_i (R_i,t_i) frame transformation between marker and tracker frames, for both poses
- F_i (R_i,t_i) takes the p_{cal} vector to the pivot point
- $F_i{*}p_{cal} = p$
- First rotation by R_i, then translation by t_i
- $R_i{*}p_{cal} + t_i = p$
- Unknowns: p_{cal} and p
- Two poses are sufficient to calculate P_{cal}

Do the math…

(1) $R_i{*}p_{cal} + t_i = p$

(2) $R_j{*}p_{cal} + t_j = p$ subtract and 1 and 2

$R_i{*}p_{cal} - R_j{*}p_{cal} + t_i - t_j = 0$

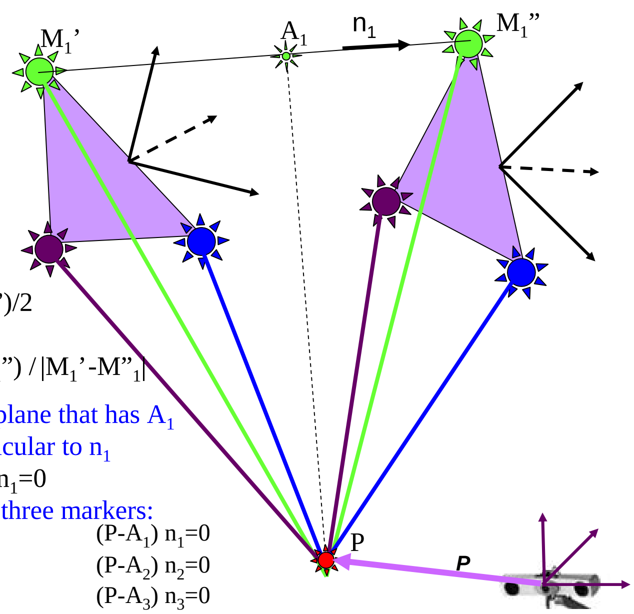
$(R_i - R_j)p_{cal} + t_i - t_j = 0$

$(R_i - R_j)p_{cal} = -(t_i - t_j)$

$\mathbf{p_{cal} = -(R_i - R_j)^{-1}(p_i - p_j)}$

- Good: one unique solution for two poses
- Bad: no measure of error. How do we know if we did anything wrong?
- Typical sources of error:
  - M_1,M_2,M_3 falsely measured by tracker → check congruency of marker-triangle
  - Tool tip slips – we will never know from 2 poses → need more poses
- Use many poses, calibrate Pcal from each of the possible n(n-1)/2 pairs, and then take the average
- Examine if there was a slip of tool tip by combinatorial examination of n(n-1)/2 cases. When Pcal changes significantly between two consecutive poses, quite likely there was a slip.

Pivot calibration w/ elementary math



$A_1 = (M_1' + M_1")/2$

$n_1 = (M_1' - M_1") / |M_1' - M"_1|$

P lays in the plane that has A_1
And perpendicular to n_1

$(P-A_1)\, n_1 = 0$

Same for all three markers:

$(P-A_1)\, n_1 = 0$

$(P-A_2)\, n_2 = 0$

$(P-A_3)\, n_3 = 0$

Pivot calibration w/ elementary math

$(P-A_1)\ n_1=0$
$(P-A_2)\ n_2=0$
$(P-A_3)\ n_3=0$

$P\ n_1=A_1n_1$
$P\ n_2=A_2n_2$
$P\ n_3=A_3n_3$

$n1=[x1\ \ y1\ \ z1]$
$n2=[x2\ \ y2\ \ z2]$
$n3=[x3\ \ y3\ \ z3]$
$P\ =[x\ \ \ y\ \ \ z\,]$

$$\begin{vmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \end{vmatrix} * \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} A_1n_1 \\ A_2n_2 \\ A_3n_3 \end{vmatrix}$$

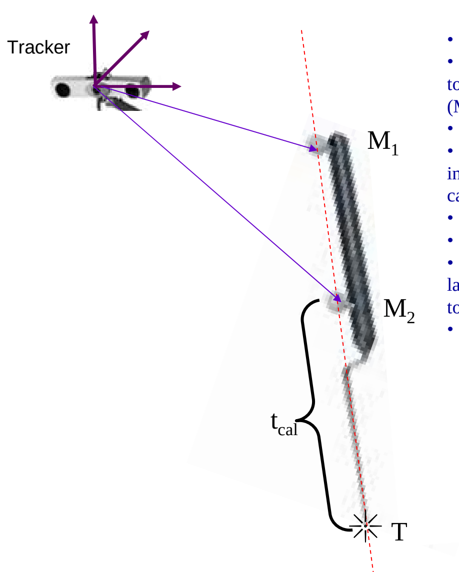
$$\boxed{\begin{vmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \end{vmatrix}^{-1} \begin{vmatrix} A_1n_1 \\ A_2n_2 \\ A_3n_3 \end{vmatrix} = \begin{vmatrix} x \\ y \\ z \end{vmatrix} = P}$$

Still must solve for p_{cal}

$R_i * p_{cal} + t_i = p$

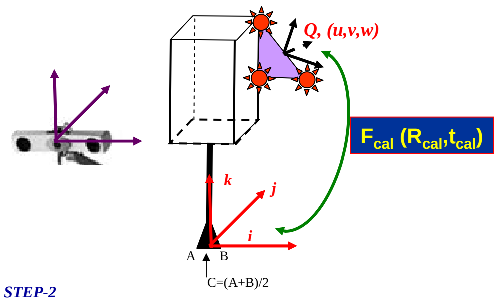
$\mathbf{p_{cal} = R_i^{-1}(p - t_i)}$

Example: Pivot calibration of a 2-marker device



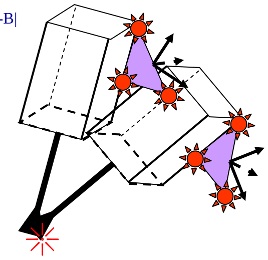
- Calibration: determine the unknown 't_{cal}'
- M1, M2 and the tool tip must be colinear, else the tool tip could not be determined from measuring (M1,M2)
- Calibration means determining the unknown 't_{cal}'
- Pivot calibration solves for T as the approximate intersection of two pivot lines in space. Then we calculate an approximate 't_{cal}'.
- Two poses are enough for calibration
- Multiple poses produces a "stronger" calibration
- Slip of the tool tip during pivoting may be caught as larger distance between two pivot line, but even this is totally fool proof, however.
- Our best is to pivot in great many directions

Example: calibration of a tracked chisel



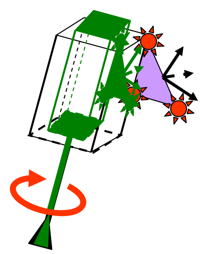
STEP-1

- Pivot about A corner of the edge
- Pivot about B corner of the edge
- C=(A+B)/2
- ***i***=(A-B)/|A-B|



STEP-2

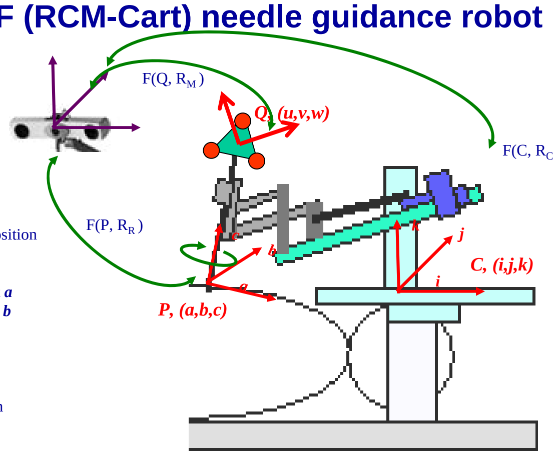
- Rotate about ***k*** axis inside a steady guide sleeve, in 3 poses
- M1 travels on a circle, calculate the center point C1
- M2 travels on a circle, calculate the center point C2
- ***k***=(C1-C2)/ |C1-C2|



STEP-3

- ***j=k x i***
- ***R_{cal} is know from (i,j,k) and (u,v,k)***
- ***t_{cal} known as t_{cal}=(C-Q)***

Tracked 5-DOF (RCM-Cart) needle guidance robot



RCM stage calibration

- Bring α and β joints to 'home' (zero) position
- Pivot needle about P → get
- Rotate about needle axis → get ***c axis***
- Rotate RCM robot about α axis 1 → get ***a***
- Rotate RCM robot about β axis 2 → get ***b***
- From (abc) get R_R
- Now we have F(P, R_R)

Cartesian stage calibration

- Bring all joints to 'home' (zero) position
- Move Cartesian stage #1 → *get **i***
- Move Cartesian stage #2 → *get **j***
- Move Cartesian stage #3 → *get **k***
- From (ijk) get R_C
- C is irrelevant

Important questions during surgery – called "Inverse Kinematics"

1. How to adjust the Cartesian robot to move needle tip from P1T to P2T (given in tracker space)? – transform v1 and v2 to Cartesian robot space and figure out translation (C will fall out of the equation!)

2. How to adjust the RCM robot to rotate needle axis from v1 direction to v2 direction (given in tracker space)? – transform v1 and v2 to RCM space and figure out rotations