Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will not be collected for grading.

Readings

Read sections 1.7, 1.8, 2.1, 2.2, 2.3, 2.4, 2.6 of Schaum’s Outline of Discrete Mathematics. Read section 2.1 of Discrete Mathematics Elementary and Beyond.

Problems

(1) Let \( \{A_i : i \in \mathbb{N}\} \) denote an arbitrary indexed class of sets. Let \( k \in \mathbb{N} \) Show that

\[
\bigcap_{i \in \mathbb{N}} A_i \subseteq A_k \subseteq \bigcup_{i \in \mathbb{N}} A_i
\]

(2) Prove using mathematical induction that the sum of the first \( n \) natural numbers is equal to \( \frac{n(n+1)}{2} \). This can also be stated as:

Prove that the proposition \( P(n) \),

\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\]

is true for all \( n \in \mathbb{N} \)

(3) Prove using mathematical induction that the proposition \( P(n) \),

\[
\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}
\]

(4) Let \( A = \{1, 2, 3\} \) and \( B = \{a, b\} \)

(a) What is \( A \times B \)?

(b) What is \( B \times A \)?

(c) What is \( (A \times B) \cup (B \times A) \)?

(d) What is \( (A \times B) \cap (B \times A) \)?

(5) Suppose \( A \) is a set of \( m \) elements, and \( B \) is a set of \( n \) elements. How many elements are there in the product set \( A \times B \)? How many elements are there in the product set \( B \times A \)?

(6) Consider the following relations on the set \( A = \{1, 2, 3\} \):

- \( R = \{(1, 1), (1, 2), (1, 3), (3, 3)\} \),
- \( S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\} \),
• \( T = \{(1, 1), (1, 2), (2, 2), (2, 3)\} \),
• \( \emptyset \),
• \( A \times A \).
Which of the relations above are reflexive, symmetric, transitive?

(7) Let \( S \) be a set of \( n \) elements, such that \( \{a, b\} \) is a subset of \( S \). What is the number of subsets of \( S \) that contain \( a \) but do not contain \( b \).