CISC-102

Fall 2015
Lecture 5
Counting

• Suppose A and B are finite sets. Then \( A \cup B \) is finite and:

\[
| A \cup B | = |A| + |B| - |A \cap B|
\]

– If A and B are not disjoint (they have common elements) then we subtract those elements in both A and B to obtain the cardinality of the union

• This is called the \textit{Inclusion-Exclusion Principle}. 
Principle of Inclusion Exclusion

Example

\[ A = \{1, 7, 22, 41\}, \quad B = \{1, 3, 9\} \]
Principle of Inclusion Exclusion
Example

A = {1, 3, 4, 6}, B = {2, 3, 4, 5}, C = {3, 5, 6, 7}
Counting

• Suppose A and B are finite sets. Then $A \cup B \cup C$ is finite and:
  
  $|A \cup B \cup C| = |A| + |B| + |C|$
  
  $-|A \cap B| - |B \cap C| - |A \cap C|$
  
  $+ |A \cap B \cap C|$
Counting

• Given a non-empty finite set $S$, the set of all subsets of $S$ is the *power set* of $S$, which we denote as $\mathcal{P}(S)$.
• $|\mathcal{P}(S)| = 2^{|S|}$
There's only 10 types of people in the world: those who understand binary and those who don't.
0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111

Counting from 0..15 in binary. We use 4 binary bits to store $16 = 2^4$ different values.
For example the binary number 0101 is

$0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 4 + 1 = 5$
There is a strong correspondence between the subsets of a 4 element set, and 4 bit binary numbers. For every subset we can define precisely one number and for every number we similarly get the same subset. We can use the term *bijection* to describe this correspondence.
0000, 0001, 0010, 0011,
∅, {4}, {3}, {3,4}
0100, 0101, 0110, 0111,
{2}, {2,4}, {2,3} {2,3,4}
1000, 1001, 1010, 1011,
{1}, {1,4}, {1,3}, {1,3,4}
1100, 1101, 1110, 1111
{1,2}, {1,2,4}, {1,2,3}, {1,2,3,4}

This table shows the bijection between subsets of {1,2,3,4}, (any 4 element set) and 4 bit binary numbers.
Counting

• Let S be a non-empty set. A *partition* of S consists of disjoint non-empty subsets of S whose union is S.
• For example: Let S be the set \( \{1,2,3,4,5,6\} \):
  • \( E = \{x \in S : x \text{ even}\} \)
  • \( O = \{x \in S : x \text{ odd}\} \)
Counting

• For example: Let $A$ be the set of integers. A partition of $A$ is:
  • $B = \{x \in \mathbb{Z}: x < 0\}$
  • $C = \{0\}$
  • $D = \{x \in \mathbb{Z}: x > 0\}$
• The subsets of a partition can be called *cells*. 
Venn Diagrams

We can also illustrate a partition of a set into cells.