Please work on these problems and be prepared to share your solutions with classmates in class next week. Assignments will not be collected for grading.

**Readings**

Read chapter 4 of *Schaum’s Outline of Discrete Mathematics*.  
Read section 3.1, 3.5 and 3.6 of *Discrete Mathematics Elementary and Beyond*.

**Problems**

(1) Prove (using mathematical induction on \( n \)) that:

\[
\sum_{m=0}^{n} \binom{m+1}{m} = \binom{n+2}{n}
\]

is true for all \( n \in \mathbb{N} \).

(2) Use a truth table to verify that the proposition \( p \lor \neg(p \land q) \) is a tautology, that is, the expression is true for all values of \( p \) and \( q \).

(3) Use a truth table to verify that the proposition \( (p \land q) \land \neg(p \lor q) \) is a contradiction, that is, the expression is false for all values of \( p \) and \( q \).

(4) Use a truth table to show that \( p \lor q \equiv \neg(\neg p \land \neg q) \)

(5) Show that the following argument is valid.

\[ p \implies q, \neg q \vdash \neg p \]

(6) Let \( A = \{1, 2, 3, 4, 5\} \). Determine the truth value of each of the following statements.

\begin{enumerate}
  \item (a) \( \exists x \in A \)( \( x + 2 = 7 \))
  \item (b) \( \forall x \in A \)( \( x + 2 < 8 \))
  \item (c) \( \exists x \in A \)( \( x + 3 < 2 \))
  \item (d) \( \forall x \in A \)( \( x + 3 \leq 9 \))
\end{enumerate}

(7) Let \( A = \{1, 2, 3, 4, 5\} \). And let \( (x, y) \in A^2 \), be the domain of the propositions given below. Determine the truth value of the following statements.

\begin{enumerate}
  \item (a) \( \exists x \forall y, x^2 < y + 1 \)
  \item (b) \( \forall x \exists y, x^2 < y + 1 \)
\end{enumerate}