Please work on these problems and be prepared to share your solutions with classmates in class next week. Assignments will not be collected for grading.

Readings
Read chapter 4 of *Schaum’s Outline of Discrete Mathematics*.
Read section 3.1, 3.5 and 3.6 of *Discrete Mathematics Elementary and Beyond*.

Problems
(1) Prove (using mathematical induction on \(n\)) that:
\[
\sum_{m=0}^{n} \binom{m+1}{m} = \binom{n+2}{n}
\]
is true for all \(n \in \mathbb{N}\).
(2) Use a truth table to verify that the proposition \(p \lor \neg(p \land q)\) is a tautology, that is, the expression is true for all values of \(p\) and \(q\).
(3) Use a truth table to verify that the proposition \((p \land q) \land \neg(p \lor q)\) is a contradiction, that is, the expression is false for all values of \(p\) and \(q\).
(4) Use a truth table to show that \(p \lor q \equiv \neg(\neg p \land \neg q)\)
(5) Show that the following argument is valid.
\[
p \rightarrow q, \neg q \vdash \neg p
\]
(6) Let \(A = \{1, 2, 3, 4, 5\}\). Determine the truth value of each of the following statements.
(a) \((\exists x \in A)(x + 2 = 7)\)
(b) \((\forall x \in A)(x + 2 < 8)\)
(c) \((\exists x \in A)(x + 3 < 2)\)
(d) \((\forall x \in A)(x + 3 \leq 9)\)
(7) Let \(A = \{1, 2, 3, 4, 5\}\). And let \((x, y) \in A^2\), be the domain of the propositions given below. Determine the truth value of the following statements.
(a) \((\exists x \forall y, x^2 < y + 1)\)
(b) \((\forall x \exists y, x^2 < y + 1)\)