CISC-102 WINTER 2020

HOMEWORK 5

Assignments will **not** be collected for grading.

Readings

Read sections 2.1, 2.2, 2.3, of *Schaum's Outline of Discrete Mathematics.*

Problems

(1) Consider the following relations on the set $A = \{1, 2, 3\}$:
- $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$,
- $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$,
- $T = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$,
- $A \times A$.

For each of these relations determine whether it is symmetric, antisymmetric, reflexive, or transitive.

(2) Explain why each of the following binary relations on the set $S = \{1, 2, 3\}$ is or is not an equivalence relation on S.
  (a) $R = \{(1, 1), (1, 2), (3, 2), (3, 3), (2, 3), (2, 1)\}$
  (b) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (3, 2), (2, 3), (3, 1), (1, 3)\}$
  (c) $R = \{(1, 1), (2, 2), (3, 3), (3, 1), (1, 3)\}$

(3) Consider a relation W on the set $\mathbb{Z}$ defined as: $W = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 = y^2\}$. Show that W is an equivalence relation. Let n be an arbitrary integer What are the elements of the equivalence class $[n]$.

(4) Let $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. We can define a relation on the powerset of A, $P(A)$, as R $= \{(X, Y) \in P(A) \times P(A) : X \cap B = Y \cap B\}$. Show that R is an equivalence relation. What is the partition of $P(A)$ with respect to R?

(5) Let R be a relation on the set of Natural numbers such that $(a, b) \in$ R if $a \geq b$. Show that the relation R on $\mathbb{N}$ is a partial order.

(6) Which of the following relations on the set $S = \{1, 2, 3, 4, 5, 6\}$ is a function?
- R = {(1,1), (2,2), (3,2), (4,2), (5,3),(6,3) }
- S = {(1,1), (2,2), (3,2), (4,2), (5,3),(6,3), (1,4) }
- T = {(1,1), (2,2), (3,3), (4,4) }
- $S \times S$