Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will not be collected for grading.

Readings

Read sections 11.1, 11.2, 11.3, 11.4, and 11.5 of *Schaum’s Outline of Discrete Mathematics*.

Read section 6.1, and 6.2 (for those wishing to delve deeper into the world of prime numbers check out sections 6.3, 6.4, and 6.5. This is definitely supplemental reading, and not a requirement of this homework set.) of *Discrete Mathematics Elementary and Beyond*.

Problems

1) Find the quotient $q$ and remainder $r$, as given by the Division Algorithm theorem for the following examples.
   (a) $a = 500$, $b = 17$
   (b) $a = -500$, $b = 17$
   (c) $a = 500$, $b = -17$
   (d) $a = -500$, $b = -17$

2) Let $a$, $b$, $c$ be Integers.
   (a) Prove that if $a|b$ and $b|c$ then $a|c$.
   (b) Prove that if $a|b$ and $a|c$, then $a|(b + c)$
   (c) Prove that if $a|b$ and $b|a$, then $|a| = |b|$, that is $a = \pm b$

3) Use induction to prove the following propositions.
   (a) $n^2 + n$ is divisible by 2, for all $n \in \mathbb{N}, n \geq 1$.
   (b) $n^3 + 2n$ is divisible by 3, for all $n \in \mathbb{N}, n \geq 1$.
   (c) $n! \geq 2^n$ for all $n \in \mathbb{N}, n \geq 4$.
   (d) $n^2 \geq 2n + 1$ for all $n \in \mathbb{N}, n \geq 3$.
   (e) $2^n \geq n^2$ for all $n \in \mathbb{N}, n \geq 4$. 

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