Please work on these problems and be prepared to share your solutions with classmates in class next week. Assignments will not be collected for grading.

Readings
Read sections 2.1, 2.2, 2.3, 11.1, 11.2, 11.3, 11.4, and 11.5 of Schaum’s Outline of Discrete Mathematics.
Read section 6.1, and 6.2 of Discrete Mathematics Elementary and Beyond.

Problems
(1) Consider the following relations on the set $A = \{1, 2, 3\}$:
- $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$,
- $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$,
- $T = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$,
- $A \times A$.
For each of these relations determine whether it is symmetric, antisymmetric, reflexive, or transitive.

(2) Explain why each of the following binary relations on the set $S = \{1, 2, 3\}$ is or is not an equivalence relation on $S$.
(a) $R = \{(1, 1), (1, 2), (3, 2), (3, 3), (2, 3), (2, 1)\}$
(b) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (3, 2), (2, 3), (3, 1), (1, 3)\}$
(c) $R = \{(1, 1), (2, 2), (3, 3), (3, 1), (1, 3)\}$

(3) Let $R$ be a relation on the set of Natural numbers such that $(a, b) \in R$ if $a \geq b$. Show that the relation $R$ on $\mathbb{N}$ is a partial order.

(4) Evaluate
(a) $|3 - 7|$
(b) $|1 - 4| - |2 - 9|$
(c) $|- 6 - 2| - |2 - 6|$

(5) Find the quotient $q$ and remainder $r$, as given by the Division Algorithm theorem for the following examples.
(a) $a = 500$, $b = 17$
(b) $a = -500$, $b = 17$
(c) $a = 500$, $b = -17$
(d) $a = -500$, $b = -17$

(6) Show that $c|0$, for all $c \in \mathbb{Z}$, $c \neq 0$. 

(7) Let \(a, b, c \in \mathbb{Z}\) such that \(c | a\) and \(c | b\). Let \(r\) be the remainder of the division of \(b\) by \(a\), that is there is a \(q \in \mathbb{Z}\) such that \(b = qa + r, 0 \leq r < |b|\). Show that under these condition we have \(c | r\).

(8) Let \(a, b \in \mathbb{Z}\) such that \(2 | a\). (In other words \(a\) is even.) Show that \(2 | ab\).

(9) Let \(a \in \mathbb{Z}\) show that \(3 | a(a + 1)(a + 2)\), that is the product of three consecutive integers is divisible by 3.

(10) Use induction to prove the following propositions.
    (a) \(n^3 + 2n\) is divisible by 3, for all \(n \in \mathbb{N}, n \geq 1\).
    (b) Show that any integer value greater than 2 can be written as \(3a + 4b + 5c\), where \(a, b, c\) are non-negative integers, that is \(a, b, c \in \mathbb{Z}, a, b, c \geq 0\).
    (c) Show that every Natural number \(n\) can be represented as a sum of distinct powers of 2. For example the number \(42 = 32 + 8 + 2 = 2^5 + 2^3 + 2^1\).