CISC-102 WINTER 2020

HOMEWORK 8

Please work on these problems and have them completed by next week. Assignments will **not** be collected for grading.

Readings

Read sections 5.1, 5.2, 5.3, 5.4, 5.5, and 5.6 of *Schaum's Outline of Discrete Mathematics.* Read section 3.1, 3.2, 3.4, and 3.5 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) How many ways are there to select a 5 card poker hand from a standard deck of 52 cards, such that none of the cards are clubs?

(2) How many ways are there to select a 5 card poker hand poker hand from a standard deck of 52 cards, such that at least one of the cards are clubs?

(3) You are planning a dinner party and want to choose 5 people to attend from a list of 11 close personal friends.
   (a) In how many ways can you select the 5 people to invite.
   (b) Suppose two of your friends are a couple and will not attend unless the other is invited. How many different ways can you invite 5 people under these constraints?
   (c) Suppose two of your friends are are enemies, and will not attend unless the other is not invited. How many different ways can you invite 5 people under these constraints?

(4) What is the number of ways to colour n different objects, one colour per object with 2 colours? What is the number of ways to colour n different objects with 2 colours, so that each colour is used at least once.

(5) Consider the equation

$$x_1 + x_2 + x_3 + x_4 = 7$$

A non-negative integer solution to this equation assigns non-negative integers (integers $x, x \geq 0$) to the variables x_1, x_2, x_3, x_4 so that the sum is 7. For example one possible solution is $x_1 = 1, x_2 = 3, x_3 = 1, x_4 = 2$. And another distinct solution is $x_1 = 2, x_2 = 3, x_3 = 1, x_4 = 1$ How many distinct non-negative integer solutions are there to this equation? How many solutions are there if only positive integers are allowed?

(6) From 100 used cars siting on a lot, 20 are to be selected for a test designed to check safety requirements. These 20 cars will be returned to the lot, and again 20 will be selected for testing for emission standards
  (a) In how many ways can the cars be selected for safety requirement testing?
  (b) In how many ways can the cars be selected for emission standards testing?
  (c) In how many different ways can the cars be selected for both tests?
  (d) In how many ways can the cars be selected for both tests if exactly 5 cars must be tested for safety and emission?

(7) Consider the equation

$$\sum_{i=0}^{2} \binom{3}{i}\binom{2}{2-i} = \binom{5}{2}. \tag{1}$$

  (a) Use algebraic manipulation to prove that the left hand and right hand sides of equation (1) are in fact equal.
  (b) Use a counting argument to prove that the left hand and right hand sides of equation (1) are in fact equal.

(8) Now consider a generalization of the previous equation.

$$\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}. \tag{2}$$

Use a counting argument to prove that the left hand and right hand sides of equation (2) are in fact equal.

(9) In the notes you will find Pascal's triangle worked out for rows 0 to 8. The numbers in row 8 are 1 8 28 56 70 56 28 8 1. Work out the values of rows 9 and 10 of Pascal's triangle with the help of the equation:

$$\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}. \tag{3}$$

(10) Show that $\binom{n}{0} = \binom{n-1}{0}$, and that $\binom{n-1}{n-1} = \binom{n}{n}$ by an algebraic argument as well as a counting argument.

(11) Prove that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0$$

Note that this equation can also be written as follows:

$$\sum_{i=0}^{n} \binom{n}{i}(-1^i) = 0$$

HINT: This can be viewed as a special case of the binomial theorem.