CISC-102 FALL 2014

HOMEWORK 8

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will **not** be collected for grading.

**Readings**

Read sections 5.3, 5.6 of *Schaum’s Outline of Discrete Mathematics.*

Read section 3.2, 3.4, 3.5 of *Discrete Mathematics Elementary and Beyond.*

**Problems**

1. We distribute \( n \) dimes to \( y \) young people and \( o \) old people in such a way that we require every young person to get at least 1 dime, but do not require the same thing for the old people. In how many ways can we do this?

2. A group of \( n \) gamblers each start out with exactly \( d \) dollars each. The rules of the game insist that all bets must be positive integer multiples of a dollar. Also no gambler can owe another any money so that in the worst case they go home with 0 dollars but no debt. How many possible results are there at the end of the gambling session.

3. Prove that

\[
\binom{n}{k} \binom{n}{n-k} = \binom{n}{k}^2
\]

4. The following question is problem 3.6.2 from *Discrete Mathematics Elementary and Beyond.* Consider the following equation:

\[
\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{2}^2 + \binom{n}{n-1}^2 + \binom{n}{n}^2 = \sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}
\]

By the binomial theorem we know that the right side of the equation is the coefficient of \( x^n y^n \) in the expansion of \((x + y)^{2n}\). Now write \((x + y)^{2n}\) as the product \((x + y)^n(x + y)^n\), and expand both factors \((x + y)^n\) using the Binomial Theorem, and then try to figure out the coefficient of \( x^n y^n \) in the product. The equation in the previous problem should be helpful. Show how this proves the equation above.

5. How many ways can a class of 12 students be partitioned into 3 groups of 4 students?
(6) How many ways can a class of 12 students be partitioned into 3 groups A, B, C, such that, there are 5 students in A, 4 students in B, and 3 students in C?

(7) How many ways can a class of 12 students be partitioned into two non-empty groups?