CISC-102 FALL 2014

HOMEWORK 9

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will not be collected for grading.

Readings

Read sections 3.1, 3.2, 3.3, 3.4 of Schaum’s Outline of Discrete Mathematics.

Problems

(1) Consider a function \( f : A \to B \) such that \( A = \{000, 001, 010, 011, 100, 101, 110, 111\} \) and \( B = \mathcal{P}(S) \) the power set of \( S = \{a, b, c\} \), and the function is as follows:

\[
\begin{align*}
(000, \emptyset), (001, \{3\}), (010, \{2\}), (011, \{2, 3\}), (100, \{1\}),
(101, \{1, 2\}), (110, \{1, 2, 3\})
\end{align*}
\]

Is this a one-to-one function, why or why not? Is this an onto function, why or why not? Is this function invertible?

(2) Let \( S \) denote a set of students. Determine which of the following relations defines a function.

(a) (student, age)
(b) (student, teacher(s))
(c) (student, pet(s))
(d) (student, student ID)

(3) Let \( X = \{1, 2, 3, 4\} \). Determine whether each of the following relations on \( X \) is a function.

(a) \( f = \{(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)\} \)
(b) \( g = \{(3, 1), (4, 2), (1, 1)\} \)
(c) \( h = \{(2, 1), (3, 4), (1, 4), (2, 1), (4, 4)\} \)

(4) Let the functions \( f \) and \( g \) be defined as \( f(x) = 2x + 1 \) and \( g(x) = x^2 - 2 \).

(a) \( (g \circ f)(4) \). That is \( g(f(4)) \).
(b) \( (f \circ g)(4) \). That is \( f(g(4)) \).
(c) \( (g \circ f)(a + 2) \). That is \( g(f(a + 2)) \).
(d) \( (f \circ g)(a + 2) \). That is \( f(g(a + 2)) \).

(5) Let \( A = \{a, b, c, d, e\} \) and let \( B \) be the set of 26 lower case letters \([a..z]\). Let \( f = \{(a, r), (b, a), (c, s), (d, r), (e, e)\} \), let \( g = \{(a, z), (b, y), (c, x), (d, y), (e, z)\} \), and let \( h = \{(a, a), (b, c), (c, e), (d, r)(e, s)\} \)

Which of these functions are one-to-one? Which of these functions are onto? Which of these functions are invertible?
(6) Consider the functions on the Reals, \( f(x) = 2x \), \( g(x) = x^3 - x \), \( h(x) = x^2 - x \).
Determine which of these functions are one-to-one, onto, invertible.

(7) Which of the following are functions on the Reals?
   (a) \( f(x) = \frac{1}{x} \)
   (b) \( f(x) = \sqrt{x} \)
   (c) \( f(x) = x^2 + 1 \)

(8) Let \( X = \{a, b\} \), and let \( Y = \{1, 2, 3\} \).
   (a) Find the number of functions from \( X \) into \( Y \).
   (b) Find the number of one-to-one functions from \( X \) to \( Y \).
   (c) Find the number of onto functions from \( X \) to \( Y \).