Seating Arrangements

There is a large table at the party and Alice wants to experience every possible seating arrangement. How many ways can 7 people sit at a table?

- Alice=1, Bob=2, Carl=3, Diane=4, Eve=5, Frank=6, George=7
Seating Arrangements

This question is equivalent to asking for the number of different ways to order 7 people.

Number of ways to order 1 person? 1.
Number of ways to order 2 people? (1,2) (2,1). 2*1
Number of ways to order 3 people? (3,1,2)(3,2,1)
(1,3,2)(1,2,3)(2,1,3)(2,3,1). 3*2*1
Number of ways to order 4 people?
Guess: 4*3*2*1= 4!
And in general there are N! ways to order N people.
For N=7, we have 7! = 5040

Seating Arrangements

This is called a permutation, and there are n! ways to get a permutation of n distinct objects.

Recall n! (n factorial) is given by the expression:

\[ n! = n(n-1)(n-2) \ldots 1 \]

\[ n! = \prod_{i=1}^{n} i \]
Factorial

\[ n! = \prod_{i=1}^{n} i \]

For large \( n \) this is a very big number. Algorithm designers need to be aware of these large numbers and avoid designing algorithms that use factorial time or space!

Lottery Tickets

Lotto 6-49, players choose 6 numbers from 1 to 49.

How many ways are there to choose to these numbers?
Lottery Tickets

Lotto 6-49, players choose 6 numbers from 1 to 49.
Simplify to 1-49. There are 49 choices.
Simplify to 2-49. 49 * 48 choices?

Simplify to 2-49. 49 * 48 choices? Tempting but wrong.
Suppose choice 1 is 42, and choice 2 is 18. That is equivalent to choice 1 is 18 and choice 2 is 42, so 49*48 double counts all possibilities. The actual answer is 49*48/2.
Lottery Tickets

For lotto 6-49, players choose 6 numbers from 1 to 49.
How many ways are there to choose to these numbers?
Solution: $49 \times 48 \times 47 \times 46 \times 45 \times 44 / 6! = 13,983,816$.
This can also be written, and pronounced 49 choose 6.

$$\binom{49}{6} = \frac{49!}{43!6!}$$