CISC-102

Fall 2014
Lecture 1

Introduction

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• Office Hours:
  Tuesday 1:00-2:00, Wednesday 3:00-4:00.
Introduction

• Class Hours

• Tuesday, 11:30-12:30  Kingston RM201 plus Kingston RM101 on quiz days
• Wednesday, 13:30-14:30  Dupuis Auditorium
• Friday, 12:30-13:30  Chernoff Auditorium
Introduction

• Homework every week. Keep up to date or you risk falling behind.
• Homework will be solved in class on due date.

Introduction

• 4 quizzes 17% each
• 1 final 32%
Motivation

• According to a recent poll (33,000 CDN students interviewed in 2014) the preferred employer is Google.
• I have been given a communication from an applicant to Google with tips on how to conduct an interview.

  #1 tip Algorithm Complexity: You need to know Big-O. If you struggle with basic big-O complexity analysis, then you are almost guaranteed not to get hired.

Motivation

• Mastering Discrete Math is the direct prerequisite to mastering algorithms and complexity.
  – Send me suggestions for applications you would like to know more about (for example, Google search, encryption, computer gaming...) and I will explain how some topic we study is related to that application
Motivation

• You should view this course as a language course. You will be learning the language of mathematics and computing!

• Math can be fun.
• Math is beautiful!
• Math is a human invention just like music, painting, sculpture, poetry, hockey, basketball, soccer, fishing …
Motivation

• The picture on the previous slide is a work of art titled “beauty”. (Prints can be purchased on-line.)

• The equation $e^{i\pi} + 1 = 0$ consists of the most important numbers in mathematics $0,1$ (integers) $\pi$ (an irrational real number) $i$ (a complex number), operations $+$ $X$ and exponentiation and the relation $\ =$.
Math is a language

A mathematical joke asks, "How many mathematicians does it take to change a light bulb?" and answers "-e^{i\pi}".
Motivation

\[ e^{i\pi} + 1 = 0 \]

Hand Shakes

- Alice is having a birthday party at her house, and has invited Bob, Carl, Diane, Eve, Frank, and George.
- They all shake hands with each other
- Q: How many handshakes?
Hand Shakes

• How many handshakes?
  – Two people shaking hands with each other counts as one handshake.
• Can you guess how many handshakes with 7 people?
  – 7 people each shakes with 6 others. So 6*7 = 42?
  – This leads to a formula for n people we would get n*n-1 handshakes.
• How about 2 people?
  – Easy: 1 handshake. But 2*1 ≠ 1?

Hand Shakes

• No? How about 2 people?
  – Easy: 1 handshake
• 3 people? 3*2=6. Correct answer is 3?
• It looks like the correct formula is 3*2/2 .
Hand Shakes

3*2 counts each handshake twice, so we divide by 2 to get the correct answer.

Hand Shakes

• How many handshakes?
  – Two people shaking hands with each other counts as one handshake.

• Can you guess how many handshakes with 7 people?
  – 7 people each shakes with 6 others.
  – So 7*6/2 = 21.

• In general we can say that for n people we have n(n-1)/2 handshakes.
Hand Shakes

• The hand shake problem seems frivolous but it is actually a representation of an important mathematical concept.

• Computer Scientists need to study these concepts so that they can properly discuss the software that they develop (and use) in terms of correctness and efficiency.

Hand Shakes

• The hand shake problem seems frivolous but it is actually a representation of an important mathematical concept.

• For example if we wanted to know which handshake was the “best” we would have to compare n(n-1)/2 of them.

• Let n = 35, 000, 000 (the population of Canada) we would have to compare 612,499,982,500,000 or roughly 612 trillion hand shakes. (Too much!)
Hand Shakes

• Let \( n = 35,000,000 \) (the population of Canada) we would have to compare 612,499,982,500,000 or roughly 612 trillion hand shakes.
• If we test one handshake per second it would take roughly 31,688 Years, 269 Days, 1 Hour. (Too long!)

Hand Shakes

• Let’s convert the hand shake problem into an “official” math problem using proper notation.
• The basic building block will be the set.
Sets

- Let’s convert the hand shake problem into an “official” math problem using proper notation.
- The basic building block will be the set.

Examples:
A={1,3,5,7,9}
B={x|x is an even integer, x>0}
C={x:x is an odd integer, x>0}
Sets

\( \mathbb{N} \) = the set of natural numbers: 1, 2, 3, \ldots
\( \mathbb{Z} \) = the set of all integers: ..., -2, -1, 0, 1, 2, ...
\( \mathbb{Q} \) = the set of rational numbers
\( \mathbb{R} \) = the set of real numbers
\( \mathbb{C} \) = the set of complex numbers

Observe that \( \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C} \).

Sets

\( \mathcal{U} \) : All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the universal set.

\( \emptyset \) : A set with no elements is called the empty set or null set.

For any set \( A \), we have: \( \emptyset \subseteq A \subseteq \mathcal{U} \)
Sets

Let $S = \{a,b,c,d,e,f,g\}$ denote the set of party goers.
A handshake can be represented as a two element subset of $S$, \{a,b\}.

Q. How many two element subsets are there of a set of $n$ elements.

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Sets

1.26 Which of the following sets are equal?

A = \{x \mid x^2 - 4x + 3 = 0\},
B = \{x \mid x^2 - 3x + 2 = 0\},
C = \{x \mid x \in \mathbb{N}, x < 3\},
D = \{x \mid x \in \mathbb{N}, x \text{ is odd}, x < 5\},
E = \{1, 2\},
F = \{1, 2, 1\},
G = \{3, 1\},
H = \{1, 1, 3\}. 