1. Let $A = \{R, G, B\}$ and $B = \{4, 2\}$

   (a) (2) What is $A \times B$?
   \[\{(R,4),(R,2),(G,4),(G,2),(B,4),(B,2)\}\]

   (b) (2) What is $(A \times B) \cap (B \times A)$?
   \[\emptyset\]

2. For each of the following binary relations on the set $S = \{1, 2, 3\}$ which is an equivalence relation on $S$ and explain why.

   An equivalence relation is reflexive, transitive, and symmetric.

   (a) (3) $R = \{(1,1), (1,2), (3,2), (3,3), (2,3), (2,1)\}$
   Not reflexive, (2,2) is missing. Not transitive, (1,3) is missing. It is symmetric. Nevertheless, it’s not an equivalence relation.

   (b) (3) $R = \{(1,1), (2,2), (3,3), (2,1), (1,2), (1,3), (2,3), (3,2)\}$
   Not symmetric, because (1,3) requires (3,1) to be present for the relation to be symmetric. It’s also not transitive because (3,2) and (2,1) requires (3,1) to be present for the relation to be transitive. It is reflexive. Nevertheless, it’s not an equivalence relation.

3. For each of the following binary relations on the set $S = \{1, 2, 3\}$ which is a partial order relation on $S$ and explain why.

   A relation is a partial order if it is reflexive, antisymmetric, and transitive.

   (a) (3) $R = \{(1,1), (1,2), (3,2), (3,3), (2,3), (2,1)\}$
   Not reflexive, because (2,2) is missing. Not transitive, because (1,2)(2,3) requires (3,1). Not antisymmetric, because both (2,3) and (3,2) are present. So it’s not a partial order.

   (b) (3) $R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$
   This is a partial order.
4. Let $a, b, c$ be Integers.

(a) (3) Prove that if $a|b$ and $b|c$ then $a|c$.
Let $p,q$ be integers, then $b = pa$ and $c = qb = qpa$.

(b) (3) Prove that if $a|b$ then for any integer $n, a|bn$.
Let $p$ be an integer, then $b = pa$ and $bn = npa$, so $a|bn$.

(c) (3) Prove that if $a|1$ then $a = \pm 1$.
Let $p$ be an integer, then $1 = pa$ and both $p,a$ are in $\{-1,1\}$.

5. Find the quotient $q$ and remainder $r$, as given by the Division Algorithm theorem for the following examples.

(a) (2) $a = 56, b = 8$
$q = 7$ and $r = 0$

(b) (2) $a = -56, b = 8$
$q = -7$ and $r = 0$

6. Consider the following relations on the set $A = \{1, 2, 3, 4\}$:

$R = \{(1,1), (1,2), (1,3), (3,3), (2,4), (1,4)\}$,

$S = \{(1,1), (1,2), (2,1), (1,4)(2,4), (4,1), (4,2), (2,2), (3,3)\}$,

$T = \{(1,1), (1,2), (2,2), (2,3)\}$,

∅, $A \times A$.

(a) (5) Which of the relations above are reflexive? Explain.
A relation is reflexive if it contains $(a,a)$ for all $a \in A$. $A \times A$ is reflexive, and the others not.

(b) (5) Which of the relations above are symmetric? Explain.
A relation is symmetric if for every $(a,b)$ there is a matching $(b,a)$. $S$, $\emptyset$, and $A \times A$ are symmetric and the others are not.

(c) (5) Which of the relations above are transitive? Explain.
A relation is transitive if $(a,b)$ and $(b,c)$ imply $(a,c)$. $R$, $\emptyset$, and $A \times A$ are transitive and $S$, and $T$ are not.
The relation $S$ is not transitive because the presence of $(4,1)$ and $(1,4)$ requires the pair $(4,4)$. The solutions used to grade the quiz had this wrong and had $S$ as transitive. Let me know if you think you that one point was removed from your score on this question incorrectly.

(d) (5) Which of the relations above are antisymmetric? Explain.
A relation $R$ is antisymmetric if $(a,b)$ in $R$ implies $(b,a)$ not in $R$. $R,T$ and $\emptyset$ are antisymmetric and the others are not.
7. (6) Prove using induction that \( n(n + 1) \) is even for all \( n \geq 1, n \in \mathbb{N} \). (That is, \( n(n + 1) \) can be written as a product \( 2q \) where \( q \in \mathbb{N} \))

Base: \( 1(1+1) = 2 \)

Induction Hypothesis: \( k(k + 1) \) is even

Induction Step: \( (k + 1)(k + 2) = k^2 + 3k + 2 = k^2 + k + 2k + 2 = k(k + 1) + 2(k + 1) \)

The first part of the sum is even by the induction hypothesis, and the second part of the sum is a multiple of 2. We also know that the sum of two even numbers is even, so we have proved that \( n(n + 1) \) is even for all \( n \geq 1, n \in \mathbb{N} \).