CISC-102
Winter 2017

Quiz 4

April 4, 2017

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. NO CALCULATORS. There are 3 pages to this quiz. Note that \(( x )\) denotes the question is worth \( x \) points.

1. (4) Here are the first 6 rows of Pascal's triangle, that is row 0 to row 5.

\[
\begin{array}{cccccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
\end{array}
\]

Write out the next 2 rows, that is, row 6 and row 7 of Pascal’s triangle using binomial coefficients.

\[
\begin{array}{cccccc}
\binom{6}{0} & \binom{6}{1} & \binom{6}{2} & \binom{6}{3} & \binom{6}{4} & \binom{6}{5} & \binom{6}{6} \\
\binom{7}{0} & \binom{7}{1} & \binom{7}{2} & \binom{7}{3} & \binom{7}{4} & \binom{7}{5} & \binom{7}{6} & \binom{7}{7} \\
\end{array}
\]
2. Prove using mathematical induction on \( m \) that
\[
\sum_{i=0}^{m} \binom{n+i}{i} = \binom{n+m+1}{m}
\]
is true for all natural numbers \( n \).

(a) (2) The base case for the proof is

**Base:**
\[
\sum_{i=0}^{1} \binom{n+i}{i} = \binom{n+1}{1} + \binom{n+1}{1} = \binom{n+2}{1}
\]

Show, by expanding the binomial coefficients that \( \binom{n}{0} + \binom{n+1}{1} = \binom{n+2}{1} \).

\[
\binom{n}{0} + \binom{n+1}{1} = 1 + n+1 = n+2 = \binom{n+2}{1}
\]

(b) (4) Now complete the proof.

**I.H.** \( \sum_{i=0}^{K} \binom{n+i}{i} = \binom{n+K+1}{K} \)

**Step:** \( \sum_{i=0}^{K+1} \binom{n+i}{i} = \sum_{i=0}^{K} \binom{n+i}{i} + \binom{n+K+1}{i} + \binom{n+K+1}{K+1} \)

\[
= \binom{n+K+1}{K} + \binom{n+K+1}{K+1}
\]

\[
= \binom{n+K+2}{K+1}
\]
3. (5) Complete the truth table below, adding columns as needed, for the proposition:

\[ p \lor \neg (p \land q). \]

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( \neg (p \land q) )</th>
<th>( p \lor \neg (p \land q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

4. (5) Consider the logical argument:

\( (p \lor q), \neg p \vdash q. \)

Complete the truth table below, adding columns as needed to determine whether the argument is valid or not. After you have completed the table explain your conclusion in a sentence or two.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( p \lor q )</th>
<th>( (p \lor q) \land \neg p )</th>
<th>( [(p \lor q) \land \neg p] \implies q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
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<td>F</td>
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<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

The truth table verifies that \( [(p \lor q) \land \neg p] \implies q \) is a tautology. Therefore, \( (p \lor q), \neg p \vdash q \) is a valid argument.