Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will not be collected for grading.

Readings
Read sections 1.1, 1.2, 1.3, 1.4, and 1.5 of Schaum’s Outline of Discrete Mathematics.
Read sections 1.1, 1.2 and 1.3 of Discrete Mathematics Elementary and Beyond.

Problems

(1) Rewrite the following statements using set notation, and then give an example by listing members of sets that match the description. For example: A is a subset of C.

\text{Answer: } A \subseteq C. \ A = \{1, 2\}, \ C = \{1, 2, 3\}.

(a) The element 1 is not a member of (the set) A.

\(1 \notin A. \ A = \{2, 4\}\).

(b) The element 5 is a member of B.

\(5 \in B. \ B = \{5, 6\}\).

(c) A is not a subset of D.

\(A \nsubseteq D. \ A = \{2, 4\} \text{ and } D = \{42, 18\}\).

(d) E and F contain the same elements.

\(E = F. \ E = F = \{7\}\).

(e) A is the set of integers larger than three and less than 12.

\(A = \{x : x \in \mathbb{Z}, 3 < x < 12\}. \ A = \{4, 5, 6, 7, 8, 9, 10, 11\}\).

(f) B is the set of even natural numbers less than 15.

\(B = \{x \in \mathbb{N}, x < 8\}. \ B = \{2, 4, 6, 8, 10, 12, 14\}\). Some books define the naturals as including 0, and 0 is a multiple of 2 so it’s even. Then \(B = \{0, 2, 4, 6, 8, 10, 12, 14\}\).

(g) C is the set of natural numbers \(x\) such that \(4 + x = 3\).

\(C = \{x : x \in \mathbb{N}, 4 + x = 3\}. \ C = \emptyset\).

(2) \(A = \{x : 3x = 6\}. \ A = 2, \text{ true or false?} \ A = \{2\}. \ A \neq 2, \text{ so the statement is false.}\)

(3) Which of the following sets are equal \(\{r, s, t\}, \{t, s, r\}, \{s, r, t\}, \{t, r, s\}\). They are all equal.

(4) Consider the sets \(\{4, 2\}, \{x : x^2 - 6x + 8 = 0\}, \{x : x \in \mathbb{N}, x \text{ is even}, 1 < x < 5\}\). Which one of these sets is equal to \(\{4, 2\}\)? They are all equal.

(5) Which of the following sets are equal: \(\emptyset, \{\emptyset\}, \{\emptyset\}\). None are equal. \(\{\emptyset\}\) is a set within a set. 0 is a number not a set, and definitely not the empty set.
(6) Explain the difference between \( A \subseteq B \), and \( A \subset B \), and give example sets that satisfy the two statements.

\( A \subseteq B \) implies that A may is a subset of B that may also be equal to A. \( A = B = \{1\} \). \( A \subset B \) implies that A is strictly a subset of B. \( A = \{1\} \), B = \{1,2\}.

(7) Consider the following sets \( A = \{1, 2, 3, 4\} \), \( B = \{2, 3, 4, 5, 6, 7\} \), \( C = \{3, 4\} \), \( D = \{4, 5, 6\} \), \( E = \{3\} \).

(a) Let \( X \) be a set such that \( X \subseteq A \) and \( X \subseteq B \). Which of the sets could be \( X \)?
For example \( X \) could be \( C \), or \( X \) could be \( E \). Are there any other sets that could be \( X \)?
\( X \) could also be \( \{2,3,4\} \).

(b) Let \( X \not\subseteq D \) and \( X \not\subseteq B \). Which of the the sets could be \( X \)?

(c) Find the smallest set \( M \) that contains all five sets.
\( M = \{1,2,3,4,5,6,7\} \)

(d) Find the largest set \( N \) that is a subset of all five sets. \( N = \emptyset \)

(8) Is an “element of a set”, a special case of a “subset of a set”?

No, an element of a set is not a subset.

(9) Phrase the handshake counting problem using set theory notation.

How many two element subsets can be chosen from an n element set.

(10) List all of the subsets of \( \{1, 2, 3\} \).
\( \emptyset \), \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,2\}, \{2,3\}, \{1,2,3\}.

(11) Let \( A = \{a, b, c, d, e\} \). List all the subsets of \( A \) containing \( a \) but not containing \( b \).
\{a\}, \{a,c\}, \{a,d\}, \{a,e\}, \{a,c,d\}, \{a,c,e\}, \{a,d,e\}, \{a,c,d,e\}