CISC-102 FALL 2018

HOMEWORK 1 SOLUTIONS

PROBLEMS

(1) Rewrite the following statements using set notation, and then give an example by listing members of sets that match the description. For example: $A$ is a subset of $C$. Answer: $A \subseteq C$. $A = \{1, 2\}$, $C = \{1, 2, 3\}$.

There are many different solutions to these questions. I have shown several possibilities.

(a) The element 1 is not a member of (the set) $A$.

\[ 1 \not\in A. \quad A = \{2, 4\}. \]

(b) The element 5 is a member of $B$.

\[ 5 \in B. \quad B = \{5, 6\} \]

(c) $A$ is not a subset of $D$.

\[ A \not\subseteq D. \quad A = \{2, 4\} \text{ and } D = \{42, 18\}. \]

(d) $E$ and $F$ contain the same elements.

\[ E = F. \quad E = F = \{7\}. \quad E \subseteq F \text{ and } F \subseteq E. \]

(e) $A$ is the set of integers larger than three and less than 12.

\[ A = \{x : x \in \mathbb{Z}, 3 < x < 12 \}. \quad A = \{4, 5, 6, 7, 8, 9, 10, 11\}. \]

(f) $B$ is the set of even natural numbers less than 15.

\[ B = \{2x : x \in \mathbb{N}, x < 8\}. \quad B = \{2, 4, 6, 8, 10, 12, 14\}. \]

(g) $C$ is the set of natural numbers $x$ such that $4 + x = 3$.

\[ C = \{x : x \in \mathbb{N}, 4 + x = 3 \}. \quad C = \emptyset. \]

(2) $A = \{x : 3x = 6\}$. $A = 2$, true or false? $A = \{2\}$. $A \neq 2$, so the statement is false.

(3) Which of the following sets are equal $\{r, s, t\}$, $\{t, s, r\}$, $\{s, r, t\}$, $\{t, r, s\}$. They are all equal. The order in which elements are written in a set is not important, unless ellipses “…” are used to denote a sequence. For example $x = \{1, 2, \ldots, 10\}$.

(4) Consider the sets $\{4, 2\}$, $\{x : x^2 - 6x + 8 = 0\}$, $\{x : x \in \mathbb{N}, x \text{ is even}, 1 < x < 5\}$. Which one of these sets is equal to $\{4, 2\}$?

They are all equal.
(5) Which of the following sets are equal: $\emptyset$, $\{\emptyset\}$, $\{0\}$. None are equal. $\{\emptyset\}$ is a set within a set. 0 is a number not a set, and definitely not the empty set.

(6) Explain the difference between $A \subseteq B$, and $A \subset B$, and give example sets that satisfy the two statements.

$A \subseteq B$ is pronounced as “$A$ is a subset of $B$” implying that $A$ is a subset of $B$ that may also be equal to $A$. $A = B = \{1\}$. $A \subset B$ is pronounced “$A$ is a proper subset of $B$” implying that $A$ is strictly a subset of $B$. $A = \{1\}$, $B = \{1,2\}$.

(7) Consider the following sets $A = \{1,2,3,4\}$, $B = \{2,3,4,5,6,7\}$, $C = \{3,4\}$, $D = \{4,5,6\}$, $E = \{3\}$.

(a) Let $X$ be a set such that $X \subseteq A$ and $X \subseteq B$. Which of the sets could be $X$? For example $X$ could be $C$, or $X$ could be $E$. Are there any other sets that could be $X$?

$X$ could also be $\{2,3,4\}$.

(b) Let $X \not\subseteq D$ and $X \not\subseteq B$. Which of the the sets could be $X$? Set A is the only set from the list that is not a subset of D and not a subset of B. There are infinitely more possibilities of sets that satisfy these requirements. For example all sets $X_i = \{x : x \in \mathbb{N}, x > 8 + i\}$ for all values of $i \in \mathbb{N}$, represents an infinite collection of sets that are not subsets of B or D.

(c) Find the smallest set $M$ that contains all five sets.

$M = \{1,2,3,4,5,6,7\}$

(d) Find the largest set $N$ that is a subset of all five sets. $N = \emptyset$

(8) Is an “element of a set”, a special case of a “subset of a set”?

No, an element of a set is not a subset.

(9) Phrase the handshake counting problem using set theory notation.

How many two element subsets can be chosen from an n element set?

(10) List all of the subsets of $\{1,2,3\}$.

$\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$.

(11) Let $A = \{a,b,c,d,e\}$. List all the subsets of $A$ containing a but not containing b.

$\{a\}$, $\{a,c\}$, $\{a,d\}$, $\{a,e\}$, $\{a,c,d\}$, $\{a,c,e\}$, $\{a,d,e\}$, $\{a,c,d,e\}$