Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will not be collected for grading.

Read sections 1.6, 1.7, 1.8 of *Schaum’s Outline of Discrete Mathematics.* Read sections 1.1, 1.2 and 1.3 again if you did not understand things last week. of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Illustrate DeMorgan’s Law \((A \cap B)^c = A^c \cup B^c\) using Venn diagrams.

(2) Write the dual of each of the following set equations.
   
   (a) \(A \cup (A \cap B) = A\)
   
   \(A \cap (A \cup B) = A\)
   
   (b) \((A \cup B) \cap (A \cap B) = \emptyset\)
   
   \((A \cap B) \cup (A \cup B) = \mathbb{U}\)
   
   (c) \(A^c \cup B^c \cup C^c = (A \cap B \cap C)^c\)
   
   \(A^c \cap B^c \cap C^c = (A \cup B \cup C)^c\)

(3) Observe that \(A \subseteq B\) has the same meaning as \(A \cap B = A\). Draw a Venn diagram to illustrate this fact.

   Draw concentric circles.

(4) Use a Venn diagram to show that if \(A \subseteq B \subseteq C\), then \(A \subseteq C\). (BONUS: Write this observation up as a proof.)

   Draw 3 concentric circles. Assume for simplicity that none of these sets are the empty set. \(A \subseteq B\) implies that every element of A is also in B, \(x \in A\) implies \(x \in B\). Similarly \(B \subseteq C\) implies that implies that every element of B is also in C, \(y \in B\) implies \(y \in C\). Thus \(A \subseteq C\).

(5) Use the Principle of Exclusion and Inclusion to show that \(|A \cup B| + |A \cap B| = |A| + |B|\).

   (It may help your understanding if you first explore an example such as \(A = \{1,2,3\}\) and \(B = \{3,4\}\).

   By PEI we have \(|A| + |B| - |A \cap B| = |A \cup B|\), the rest follows by arithmetic.

(6) What are the cardinalities of the following sets?

   (a) \(A = \{\text{winter, spring, summer, fall}\}\). \(|A| = 4\).

   (b) \(B = \{x : x \in \mathbb{Z}, 0 < x < 7\}\). \(|B| = 6\).

   (c) \(P(B)\), that is, the power set of B. \(|P(B)| = 2^6 = 64\).

   (d) \(C = \{x : x \in \mathbb{N}, x \text{ is even}\}\) This set has infinitely many elements.
Figure 1. The top diagram illustrates $(A \cap B)$, the bottom two are $A^c$ and $B^c$, and the second from the top is to illustrate that $(A \cap B)^c = A^c \cup B^c$.
(e) \( D = \{ \text{students currently registered at Queen’s University}\} \) 24,582 (Fall 2011) 17,800 3,900 2,700 450 = 24850

(f) \( E = \{ \text{students currently registered in CISC-102}\} \) 173

(7) Suppose that we have a sample of 100 students at Queen’s who take at least one of the following language courses, French-101, Spanish-101, German-101. Also suppose that 65 take French-101, 45 take German-101, 42 take Spanish 101, 20 take French-101 and German-101, 25 take French-101 and Spanish-101, and 15 take German-101 and Spanish-101.

(a) How many students take all three language courses?
(b) Draw a Venn diagram representing these 100 students and fill in the regions with the correct number.
(c) How many students take exactly 1 of these courses?
(d) How many students take exactly 2 of these courses?

![Venn Diagram](image)

**Figure 2. Language Courses Venn Diagram**

(8) Let \( S = \{ a,b,c,d,e,f,g \} \). Determine which of the following are partitions of \( S \):

(a) \( P_1 = \{ \{a,c,e\},\{b\},\{d,g\} \} \) No because \( f \) is missing

(b) \( P_2 = \{ \{a,b,e,g\},\{c\},\{d,f\} \} \) Yes.

(c) \( P_3 = \{ \{a,e,g\},\{c,d\},\{b,c,f\} \} \) No because sets are not disjoint.

(d) \( P_4 = \{ \{a,b,c,d,e,f,g\} \} \)

(9) Recall that the union operation is associative, that is \( A \cup (B \cup C) = (A \cup B) \cup C \). Show that the relative complement set operation is not associative, that is, \( A \setminus (B \setminus C) = \)
(A\B)\C, is incorrect for some sets A, B, C. (Note if relative complement is associative then the equation must be true for all sets A, B, C.)

Let A = \{1,2,3\} B = \{1,2\} and C = \{2,3\}. A\(B\cap C) = \{2,3\} and (A\B)\C = \emptyset

(10) I will use induction (incorrectly) to prove that n(n+1) is odd for very natural number n. Proof: Assume that (k-1)k is odd. Then k(k+1) = k^2 + k = k^2 - k + 2k = (k - 1)k + 2k. So (k-1)k is odd by the induction hypothesis and 2k is even. The sum of an odd number plus an even number is always odd, thus we have proved that n(n+1) is odd for every natural number.

What is the flaw in this proof. (Hint: It’s not the arithmetic or the algebra.)

The base case is missing.

(11) BONUS1: First read section 1.3 of Discrete Mathematics Elementary and Beyond in particular the two proofs of theorem 1.3. Now show that a nonempty set with an odd number of elements has the same number of subsets with an odd number of elements as those with an even number of elements.

Use the bit representation for sets. For every representation with an even number of 0’s and an odd number of 1’s there is a partner set with an odd number of 0’s and an even number of 1’s.

(12) BONUS2: Show that a nonempty set with an even number of elements has the same number of subsets with an odd number of elements as those with an even number of elements.

Remove an arbitrary element, x, from S to get an odd number of elements. Then using the result of the previous question we get all subsets of S that do not include x. This collection has the same number of even and odd subsets. Now create an equal number of sets by adding the x. These too will have the same number of even and odd subsets.

Both questions can be answered with one proof using the binary tree method for counting the number of subsets of a set.